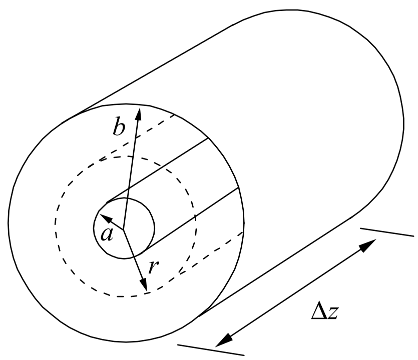
Why is 50Ω Coaxial Line so Special Anyway?

Field Analysis of Coax:

The coaxial line segment shown above is filled with a dielectric ϵ, and is assumed to be driven by a potential difference V between the inner and outer conductors, which induces a charge $\pm Q$ on the surface of each conductor. The charge will be distributed uniformly along the length Δz of the coax.



The electric field will be radial by symmetry. From Gauss' law

$$\oiint \epsilon \overline{E} \cdot d\overline{S} = \iiint \rho \, dV \quad \Rightarrow \quad E_r = \frac{Q}{2\pi\epsilon r \Delta z} \tag{1}$$

Which gives a voltage

$$V = -\int_b^a \overline{E} \cdot d\ell = \frac{Q}{2\pi\epsilon\Delta z} \ln(b/a) \tag{2}$$

The capacitance per unit length is given by

$$C = \frac{Q}{V\Delta z} = \frac{2\pi\epsilon}{\ln(b/a)} \quad [\text{F/m}] \tag{3}$$

From Ampère's law,

$$\oint \overline{H} \cdot d\ell = I_0 \quad \Rightarrow \quad H_\phi = \frac{I_0}{2\pi r} \tag{4}$$

The inductance per unit length is defined by

$$L = \frac{1}{I\Delta z} \iint \overline{B} \cdot d\overline{S} = \frac{\mu}{2\pi} \ln(b/a) \quad [\text{H/m}] \tag{5}$$

The characteristic impedance is therefore

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{\eta}{2\pi} \ln(b/a) \tag{6}$$

Power Handling Capacity:

Dielectric breakdown will occur in the region between the two conductors if the electric field exceeds a certain critical value. The field strength is a function of the applied voltage and line geometry. Using (1) and (2) we can express the electric field as

$$E_r = \frac{V}{r \ln(b/a)} \tag{7}$$

This shows that the field is largest near the center conductor, so

$$E_{max} = \frac{V}{a \ln(b/a)} \tag{8}$$

The peak power transmitted down the line is then given by

$$P = V^2/Z_0 = \frac{2\pi}{\eta} a^2 E_{max}^2 \ln(b/a) \tag{9}$$

and thus the maximum power flow is influenced by the line geometry. To find the optimum conductor sizes, we can look for the value of a which maximizes (9)

$$\frac{\partial P}{\partial a} \propto \frac{\partial}{\partial a}\left(a^2 \ln b - a^2 \ln a\right) = a\left[2\ln(b/a) - 1\right] = 0 \tag{10}$$

This equation is satisfied when $b/a = 1.65$, which gives an optimum characteristic impedance of $Z_0 = 30\,\Omega$ for maximum power transmission in a coaxial air-line.

Attenuation:

From the distributed circuit model for a transmission-line, we found that the attenuation constant (for low-loss lines) is

$$\alpha \approx \frac{R}{2Z_0} + \frac{GZ_0}{2} \tag{11}$$

where R is the series resistance per unit length, and G is the shunt conductance per unit length. Physically, where does this loss come from? The series resistance R comes from Ohmic losses in the metal conductors. Using a sheet resistivity of R_s, the then total resistance per unit length is just

$$R = \frac{R_s}{2\pi}\left(\frac{1}{b} + \frac{1}{a}\right) \tag{12}$$

The shunt conductance comes from loss in the dielectric material. If the dielectric has a small conductivity σ, then a small current can flow radially through the material according to $J_r = \sigma E_r$. The total conduction current through the dielectric is then

$$I_d = 2\pi r \Delta z J_r = 2\pi r \Delta z \sigma E_r \tag{13}$$

Using (7), the conductance G is expressed as

$$G = \frac{I}{V\Delta z} = \frac{2\pi\sigma}{\ln(b/a)} \tag{14}$$

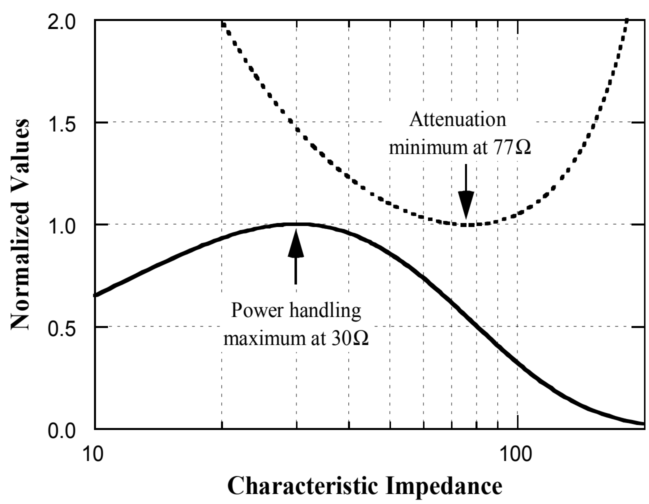
Substituting (12) and (14) into (11), we can find the optimum line dimensions for lowest attenuation,

$$\begin{aligned} \frac{\partial \alpha}{\partial a} = 0 \;\propto\; \frac{\partial}{\partial a}\frac{(1/b + 1/a)}{\ln(b/a)} \\ 0 = 1 + a/b - \ln(b/a) \end{aligned} \tag{15}$$

This equation is satisfied for $b/a = 3.6$, which gives an optimum characteristic impedance of $Z_0 = 77\,\Omega$ for lowest attenuation in a coaxial air-line.

A Compromise:

The expressions for attenuation and power handling are plotted below as a function of characteristic impedance for a coaxial air-line. An impedance of around $50\,\Omega$ gives the best overall performance for an air-dielectric. Note, however, that filling the coax with a dielectric material (such as PTFE, $\epsilon_r \approx 2.25$) will shift the optimum points to a lower characteristic impedance.*



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