MODELLING NEWTONIAN AND RELATIVISTIC NEUTRON STARS

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Neutron stars are fascinating astrophysical objects. Their general characteristics are predicted by the theory of stellar evolution, for which there is abundant observational evidence (pulsars, magnetars and binary systems). This work aims to compare the properties of neutron stars obtained through different models. We begin our study of neutron stars with Newtonian stellar models using polytropical equations of state (EoS). The internal structure of the polytropes is obtained by the numerical integration of the Lane-Emden equation. For the several conditions simulated (values of polytropic index, mass, radius, temperature, pressure and density), fundamental properties of Newtonian stars were obtained such as the Chandrasekhar's limit for white dwarfs. However, Newtonian equations cannot correctly describe very compact and massive objects. There is an upper mass limit for neutron stars, supported by observations, which is not predicted by Newtonian equations. Neutron stars are best described under the framework of General Relativity. The introduction of TOV equations (as well as relativistic Lane-Emden equations) in the relativistic treatment is, therefore, necessary to correctly identify stable and non-stable models (mass-radius relation). The analysis of the EoS also becomes relevant to prevent a possible violation of causality (sound speed larger than c).

Keywords: Neutron Stars, Newtonian and Relativistic Stars, General Relativity, Causality, Stability.

INTRODUCTION

The current period of discoveries in the fields of Physics allows the acquisition of knowledge and understanding of physical phenomena previously unknown. Cosmology, for instance, is no different. The accelerated expansion of the universe, the recurrent related terms energy and dark matter, the possible emission of gravitational waves in binary systems among others are the new phenomenology present nowadays. Amongst this phenomenological diversity, one can also highlight an object of fundamental importance to the physical verification of gravitational waves emission: neutron stars. Thus, the study of these compact objects is important for the substantiation of predictions made based on the theory of General Relativity[1]. Accurate models of neutron stars are still under development. The equation of state of matter in extreme conditions found in the center of those objects remains unknown. There is a variety of factors that can be taken into account. The existence of a solid crust, magnetic fields, rotation and superfluidity are just a few[1]. The study of neutron stars is therefore highly interdisciplinary, covering many areas of Physics. Nevertheless, it is necessary to check whether models derived from different assumptions and simplifications are physically consistent and correct. Due to its high compactness it is natural that neutron stars are studied under the framework of General Relativity.

OBJECTIVES

The main scientific goal of this project is to obtain a comparison of the properties of neutron stars obtained from different models, investigating thermodynamic and mechanical properties. The analytical study begins with Newtonian models and the subsequent inclusion of relativistic corrections. Elaboration of a program for numerical integration of the equations of structure, starting initially with Excel and subsequent transition to MATLAB®.

METHODOLOGY

We use analytical polytropic equations of state (Eq.1). The resolution of the equations has been obtained from standard numerical methods. For the initial simulation of Newtonian stars we used both the structure (Eq.2) and the Lane-Emden equations (Eq.3), adopting Euler's numerical method and software Excel. The next procedure was the usage of MATLAB® to implement the numerical method of Runge-Kutta of 4th order to obtain greater precision in the integration of systems of coupled differential equations.

$$P = K\rho^{\gamma} \quad (1)$$

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}\,, \quad m(r) = \int_0^r \rho 4\pi r^2 dr \quad (2)$$

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) + \theta^n = 0 \quad (3)$$

In the relativistic treatment, it is relevant to consider and adopt the Tolman-Oppenheimer-Volkoff (TOV) (Eq.4) and the Lane-Emden relativistic equations (Eq.5). They introduce relativistic corrections for a better model of massive and compact objects behavior.

$$-\frac{dP}{dr} = G\frac{(m + 4\pi r^3 P/c^2)}{r^2(1 - 2Gm/c^2 r)}\left(\rho + \frac{P}{c^2}\right), \quad m(r) = \int_0^r \rho 4\pi r^2 dr \quad (4)$$

$$\frac{dv}{d\xi} = \xi^2\theta^n\,, \quad -\xi^2\frac{d\theta}{d\xi} = \frac{(v + \sigma\theta\xi dv/d\xi)(1 + \sigma\theta)}{1 - 2\sigma(n+1)v/\xi} \quad (5)$$

RESULTS

The modeling process, in this project, consisted in the use of numerous stellar characteristic conditions represented by a polytropic fluid, varying central values and EoS, allowing different simulations of stellar interiors and obtaining different values of mass, temperature, pressure and density behavior[2]. The case for which $n = 3$ represents the stellar standard model. This value applies to the relativistic degenerate case. The results for the simulation of a star of polytropic index $n = 3$ are shown in FIG. 1 and were based on a star with input parameters 2.5 M_{Sun} and radius of 10 km. Starting from fixed values of n and κ and varying the stellar central density ρ_c, it is intended to verify the behavior of Newtonian stars and discover maximum masses. For $n = 3$, the mass is independent of the central density ρ_c. This implies that mass is constant for all radii values.

In addition, the behavior of central density ρ_c changes for different values of radius. As stellar radius is increased, ρ_c decreases. For higher values of ρ_c, more massive the stars would be. Hence, more compact they should be, so that pressure is able to withstand gravitational collapse.The independence of mass in relation to the values of ρ_c and, therefore, its constancy for different radii values denote the obtention of a maximum mass. This limit indicates stars governed by $n = 3$ cannot assume greater mass values; otherwise it will suffer gravitational collapse[1,3]. Thus, it is possible to simulate the behavior of stellar masses for different values of κ in order to obtain the Chandrasekhar limit, the maximum mass for white dwarfs (1.4 M_{Sun}). We can infer based on the results that the value of κ that best fits the Chandrasekhar limit for white dwarfs is 4.78999×10^{14} $cm^3 / (g^{1/3}s^2)$. Simulations involving different polytropic indices other than $n = 3$, masses tend to grow indefinitely in the Newtonian case, whereas General Relativity predicts that there is a maximum value of mass for each equation of state.

For the relativistic treatment, we have extended our calculations, including all results previously done, to stiffer (incompressible matter, $n \to 0$) EoS, searching for maximum masses for each value of polytropic index. From this, it was possible to verify if all solutions were subluminal and stable.

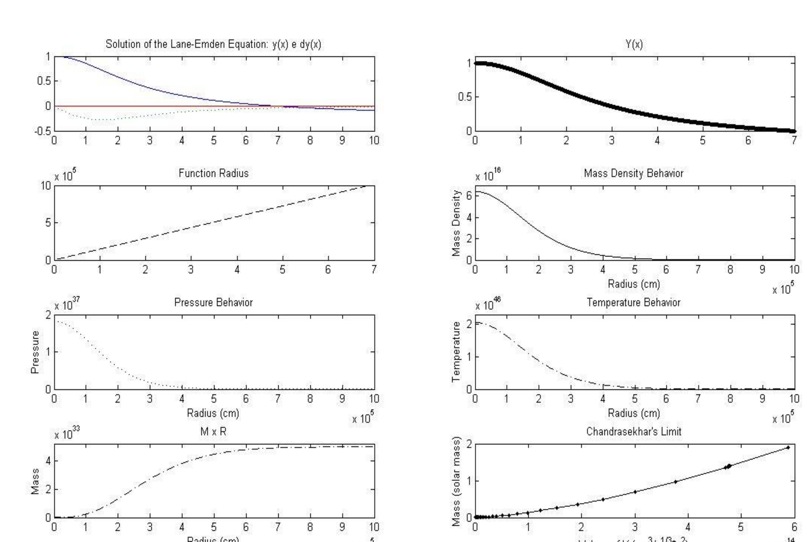

FIG. 1 – Simulation for n = 3. Solutions for the non-relativistic Lane-Emden Equation. Density, Pressure, Temperature and Mass behavior

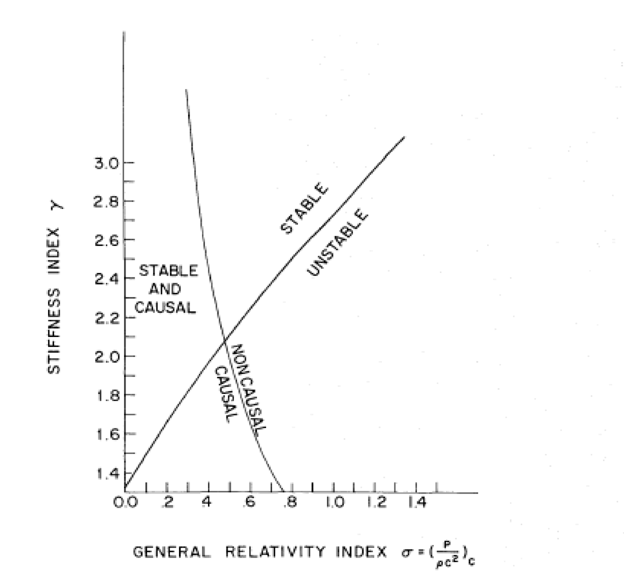

FIG. 2 – Critical value of the G.R. index, above which a polytrope is destabilized by G.R. effects. Regions in which the EoS at the star's center is causal and non-causal are also indicated[4].

The general-relativity index $\sigma = \left(\frac{P}{\rho c^2}\right)$ can also be written as an approximation $\sigma \sim GM/c^2R$. This implies the adequacy of a first post-Newtonian approximation. In G.R., for a given value of σ, the minimum value of γ necessary for stability is raised above the value 4/3 which would be sufficient for stability in Newtonian theory[4]. Conversely, a polytrope of given exponent γ or polytropic index n becomes unstable when the dimensionless general-relativity index σ exceeds σ_{CR}. Also drawn on FIG. 2 is the causality limit curve . This intersects σ_{CR} curve at $\sigma_{CR} = 0.48$, where $\gamma = 2.084$, $n = 0.926$. The region of high γ and low σ for which the EoS is causal and the star is stable is marked. The maximum value of σ is set by general relativity for $\gamma < 2.084$ and by causality for $\gamma > 2.084$.

CONCLUSIONS

The present work enabled the acquisition of fundamental knowledge to develop models of Newtonian and Relativistic stars. Through the implementation of numerical methods along with literature review, it was possible to obtain results that simulate the intra-stellar medium and to verify the main physical properties of polytropic stars. As for the simulations, the results obtained represent a good model when compared to previous results found in literature. Nonetheless, the verification that the Newtonian equations do not properly explain the behavior of stellar masses for different radii has become essential. Only under the framework of General Relativity, it is possible to correctly describe very compact and massive objects such as neutron stars.

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