A Brief Analysis of Criteria for Blind Deconvolution of Temporally-Correlated Sources

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Abstract – In the context of temporally correlated sources, blind deconvolution can be performed using the well-known constant modulus algorithm and, most recently, also using approaches based on information theory, like correntropy. However, given the different nature of both classes, a certain degree of discrepant behavior is expected when they are compared. In order to make a bridge between both criteria, we propose an autocorrelation-based criterion that is directly applicable to scenarios with non-i.i.d. sources. The associated algorithm will work as a comparative performance tool for the presented methods. We also show a brief analysis of the exposed criteria and their performance considering correlated (or coded) sources for different channel and noise models, having in view, especially, the attainable levels of intersymbol interference.

Keywords: *Blind deconvolution, correlated sources, correntropy, constant modulus criterion.*

1. Introduction

Blind deconvolution (or equalization) methods have been a major focus of attention, in last decades, in the field of signal processing. Their incontestable importance lies on the fact that they depend solely on the statistical characteristics of the transmitted signal and the channel output, i.e., a reference signal is not necessary. Considering classical techniques such as the *constant modulus (CM)* [1] criterion, a good performance is guaranteed when the transmitted signal is composed of independent samples. However, when making use of error-correcting codes or temporally correlated sources, this condition is no longer valid, and these algorithms may present a non-satisfactory performance [2].

In view of this, a recent approach, based on the field of *information theory (IT)*, was proposed by Santamaria et al. [3], in which the temporal structure of the transmitted signal is considered. As the conception deals with a variant of source correlation, it received the name of *correntropy*.

This new perspective opened the door to a better understanding of blind equalization criteria when a temporally-correlated signal is transmitted. A step in that sense was made by Neves et al. [4], which presented elements of a comparison between the CM algorithm (CMA) and a correntropy-based method. Given the differences between these methods, discrepant behavior is expected, to a certain extent, whenever a comparison be made. In view of this, we try to bridge the gap between them by presenting another element for comparison. This new element is an autocorrelation-based criterion that is capable of handling correlated sources and can serve as an interesting counterpart to correntropy-based solutions.

2. Blind Equalization Criteria: The Constant Modulus and The Correntropy

The *constant modulus (CM) criterion* has been widely studied since Godard's proposal [1]. It is based on the idea of minimizing a dispersion of the absolute value of the equalizer output around a fixed value that depends on statistics of the transmitted signal, which gives rise to the following cost function:

$$J_{CM}(\mathbf{w}) = E\left[\left(|y(n)|^2 - R_2\right)^2\right] \tag{1}$$

where $R_2 = E\left[|s(n)|^4\right] / E\left[|s(n)|^2\right]$, *y(n)* is the equalizer output signal and *s(n)* is the transmitted signal. Its gradient is given, in the case of a linear finite impulse response (FIR) equalizer by:

$$\nabla J_{CM}(\mathbf{w}) = \left(|y(n)|^2 - R_2\right) y(n)\mathbf{x}(n) \tag{2}$$

An analysis of the structure of Eq.(1) has been carried out considering, as a rule, that the transmitted signal is composed of independent and identically distributed (i.i.d.) samples. This assumption is valid for certain applications, but not for others e.g. those including error-correcting codes or even those associated with certain non-digital signals.

Another technique was recently proposed, based on IT and kernel methods, which consists of a generalized correlation function to which the name *correntropy* was associated [3]. It is possible to define correntropy as $\hat{V}[m] = \frac{1}{N-m+1}\sum_{n=m}^{N}\kappa(x(n)-x(n-m))$, where $\kappa(\cdot)$ denotes a kernel function, N is the size of the data window used to estimate correntropy and m is the lag being considered. The used criterion is stated as follows:

$$J_C(\mathbf{w}) = \sum_{m=1}^{P}(V_s[m]-V_y[m])^2 \tag{3}$$

where V_s [.] is the correntropy of the source, V_y [.] is the correntropy of the equalizer output and P is the number of lags. The associated gradient of $J_C(\mathbf{w})$ can be expressed as

$$\nabla J_C(\mathbf{w}) = \sum_{m=1}^{P}\left[(V_s[m]-V_y[m])\left(\frac{-1}{N-m}\sum_{i=n-N-m+1}^{n}\kappa(y(i)-y(i-m))(y(i)-y(i-m))(\mathbf{x}(i)-\mathbf{x}(i-m))\right)\right] \tag{4}$$

Iterative algorithms associated with both criteria can be obtained by resorting to the stochastic gradient approach.

3. Correlated Sources: Correlation Retrieval Criterion

Since the CM criterion does not explicitly promote the recovery of the source correlation profile, the correntropy criterion Eq. (3) presents advantages in non-i.i.d. scenarios. Hence, in order to provide more adequate bases for comparison with correntropy, we propose the use of the autocorrelation function, defined as *r[k] = E[y(n) y*(n-k)]*.

In analogy with what is done when correntropy is employed, we shall build an approach that tries to recover the transmitted source correlation by using exclusively the autocorrelation function $r[k]$. Based on Eq. (3), we propose the following second-order statistics criterion, named Correlation Retrieval (CR) criterion:

$$J_{CR}(\mathbf{w}) = \sum_{k=0}^{l_k}(r_y[k]-r_s[k])^2 \tag{5}$$

whose gradient is given by:

$$\nabla J_{CR}(\mathbf{w}) = \sum_{k=0}^{l_k}\left(r_y[k]-r_s[k]\right)\left(\mathbf{x}(n)y^*(n-k)+y(n)\mathbf{x}^*(n-k)\right) \tag{6}$$

where l_k is the number of considered lags. The gradient of CR criterion is considerably simpler than that of correntropy Eq. (4), since it does not makes use of kernel functions and also presents only cross delayed product terms between $x(n)$ and $y(n)$. Furthermore, if we look carefully at Eq. (5), we will see that, for $l_k = 0$ and assuming $r_s[0] = 1$, the CR criterion is very similar to the CM criterion, Eq. (1), differing only by the expectation operator which acts just over the signal $y(n)$. Also, in this case, considering an online algorithm that takes only instantaneous statistics, the gradients of CM and CR, Eq. (2) and Eq.(6) respectively, coincide.

As seen so far, it is possible to see that the CR algorithm (CRA) – obtained by using the stochastic gradient approach – although based on the correntropy criterion, also has points of contact with the classical CMA. From this perspective, a comparison involving the performance of the three criteria seems very interesting, in a similar way as that presented in [4].

4. Results

In simulation tests, we compare the three aforementioned criteria in terms of intersymbol interference (ISI). In the first case, the scenario is composed of an alternate mark inversion (AMI) source signal, a correlated sequence drawn from the alphabet {-1,0,+1}, transmitted through a channel with transfer function $h(z) = 1 + 0.6z^{-1}$. Fig. 1a shows the ISI performance for an average of 25 independent simulations with a signal to noise ratio (SNR) of 20 dB. The CMA step-size parameter was 0.001; for CRA, the parameters were $\mu_{CR}=0.002$ and $l_k=4$, and finally, correntropy was simulated considering $P=5$, $N=100$, $\sigma=1.8$ and $\mu_{corr}=0.1$. Equalizers with 3 taps were initialized using the center spike method. In this case, CM performed poorly, while correntropy seems to be trapped in a local minimum. The CRA had the best performance in this scenario, which might be related to a more favorable local minima configuration.

In the second case, we consider a precoder with transfer function $g(z) = 1 + 1.5z^{-1}$, a channel with $h(z) = g(z)$ and impulsive noise. The chosen parameters were $\mu_{CMA}=3\times10^{-5}$, $\mu_{CR}=6\times10^{-5}$, $l_k=5$, $P=6$, $N=100$, $\sigma=1.6$ and $\mu_{corr}=0.06$. Fig. 1b illustrates averaged performance of 25 simulations with 3-tap equalizers with center-spike initialization and SNR level of 20 dB. It is possible to see that CRA, in spite of a slower convergence, was able to achieve a performance similar to that of correntropy.

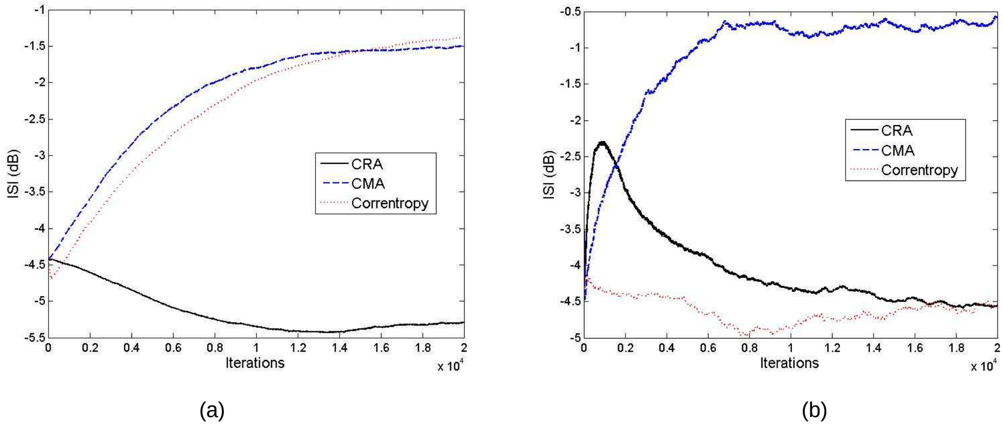

Figure 1. ISI Performance of the criteria for (a) AMI source, channel $h(z) = 1 + 0.6z^{-1}$, SNR=20dB and (b) precoded source, channel $h(z) = 1 + 1.5z^{-1}$, impulsive noise with SNR=20dB.

5. Conclusion

The use of the CRA seems to be relevant for comparing correntropy-based algorithms and the CMA in scenarios for which the sources are non-i.i.d. Simulations suggest that, given the exposed scenarios, the relatively simple form of the CR criterion – based solely on second order statistics - can give rise to an interesting local optima configuration while attaining a sound performance.

Further work will be conducted in order to analyze the criteria and also potential relationships with second-order methods for source separation.

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