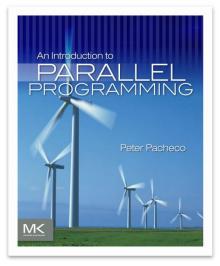


An Introduction to Parallel Programming

Peter Pacheco



#### Chapter 5

### Shared Memory Programming with OpenMP



#### Roadmap

- Writing programs that use OpenMP.
- Using OpenMP to parallelize many serial for loops with only small changes to the source code.
- Task parallelism.
- Explicit thread synchronization.
- Standard problems in shared-memory programming.

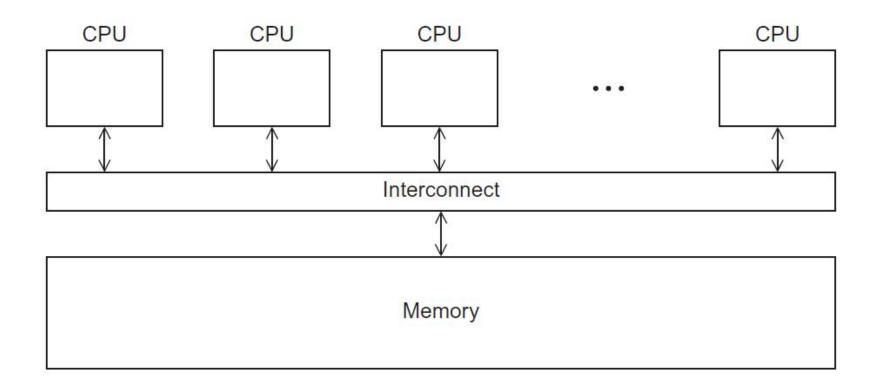


#### **OpenMP**

- An API for shared-memory parallel programming.
- MP = multiprocessing
- Designed for systems in which each thread or process can potentially have access to all available memory.
- System is viewed as a collection of cores or CPU's, all of which have access to main memory.



#### A shared memory system





#### **Pragmas**

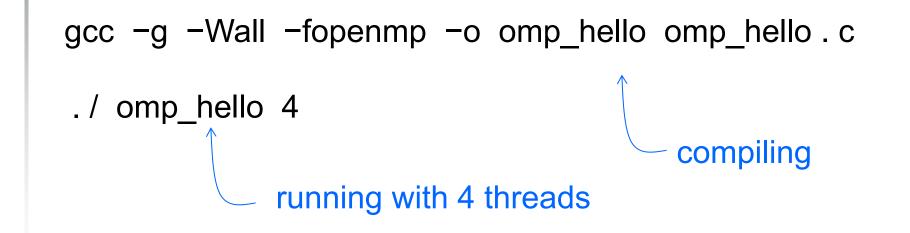
- Special preprocessor instructions.
- Typically added to a system to allow behaviors that aren't part of the basic C specification.
- Compilers that don't support the pragmas ignore them.

#pragma

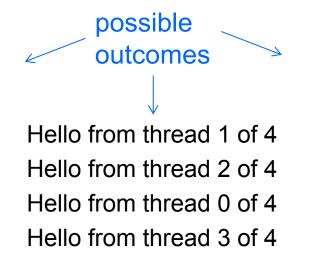


```
#include <stdio.h>
#include <stdlib.h>
#include <omp.h>
void Hello(void); /* Thread function */
int main(int argc, char* argv[]) {
   /* Get number of threads from command line */
   int thread_count = strtol(argv[1], NULL, 10);
#
  pragma omp parallel num_threads(thread_count)
   Hello();
   return 0;
}
  /* main */
void Hello(void) {
   int my_rank = omp_get_thread_num();
   int thread count = omp get num threads();
   printf("Hello from thread %d of %d\n", my_rank, thread_count);
  /* Hello */
```





Hello from thread 0 of 4 Hello from thread 1 of 4 Hello from thread 2 of 4 Hello from thread 3 of 4



Hello from thread 3 of 4 Hello from thread 1 of 4 Hello from thread 2 of 4 Hello from thread 0 of 4

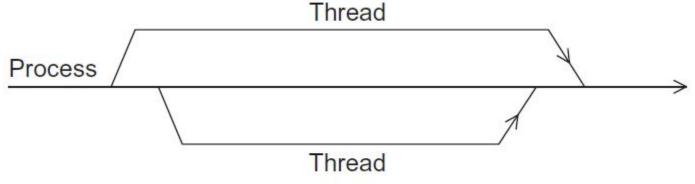


#### **OpenMp pragmas**

- # pragma omp parallel
  - Most basic parallel directive.
  - The number of threads that run the following structured block of code is determined by the run-time system.









#### clause

- Text that modifies a directive.
- The num\_threads clause can be added to a parallel directive.
- It allows the programmer to specify the number of threads that should execute the following block.

# pragma omp parallel num\_threads ( thread\_count )



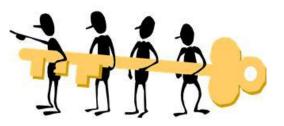
### Of note...

- There may be system-defined limitations on the number of threads that a program can start.
- The OpenMP standard doesn't guarantee that this will actually start thread\_count threads.
- Most current systems can start hundreds or even thousands of threads.
- Unless we're trying to start a lot of threads, we will almost always get the desired number of threads.



#### **Some terminology**

In OpenMP parlance the collection of threads executing the parallel block — the original thread and the new threads — is called a team, the original thread is called the master, and the additional threads are called slaves.





## In case the compiler doesn't support OpenMP

# include <omp.h>

#ifdef \_OPENMP
# include <omp.h>
#endif



# In case the compiler doesn't support OpenMP

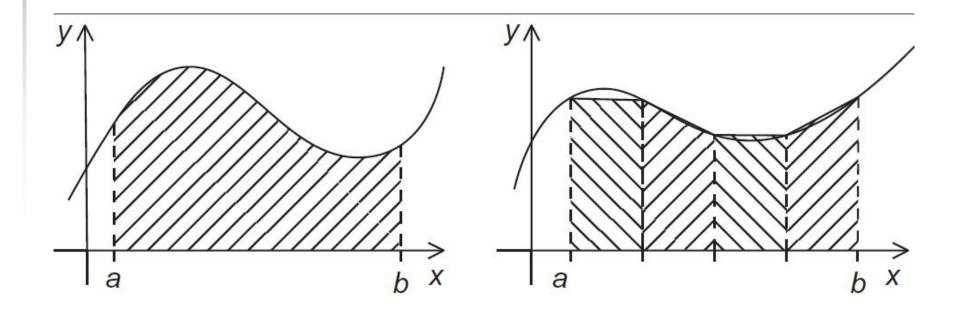
```
# ifdef OPENMP
 int my rank = omp get thread num ();
 int thread count = omp get num threads ();
#else
 int my rank = 0;
 int thread count = 1;
# endif
```



## THE TRAPEZOIDAL RULE



#### The trapezoidal rule





#### **Serial algorithm**

```
/* Input: a, b, n */
h = (b-a)/n;
approx = (f(a) + f(b))/2.0;
for (i = 1; i <= n-1; i++) {
    x_i = a + i*h;
    approx += f(x_i);
}
approx = h*approx;</pre>
```



#### **A First OpenMP Version**

1) We identified two types of tasks: a) computation of the areas of individual trapezoids, and b) adding the areas of trapezoids. 2) There is no communication among the tasks in the first collection, but each task in the first collection communicates with task 1b.



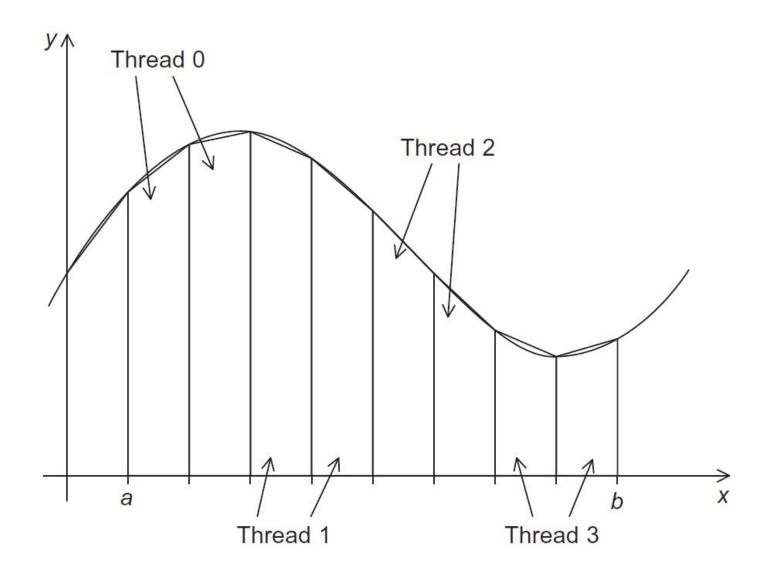
#### **A First OpenMP Version**

3) We assumed that there would be many more trapezoids than cores.

So we aggregated tasks by assigning a contiguous block of trapezoids to each thread (and a single thread to each core).



#### **Assignment of trapezoids to threads**

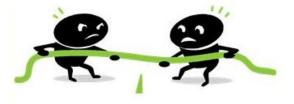




Time	Thread 0	Thread 1
0	global_result = 0 to register	finish my_result
1	<pre>my_result = 1 to register</pre>	global_result = 0 to register
2	add my_result to global_result	<pre>my_result = 2 to register</pre>
3	<pre>store global_result = 1</pre>	add my_result to global_result
4		<pre>store global_result = 2</pre>

Unpredictable results when two (or more) threads attempt to simultaneously execute:

global\_result += my\_result ;





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# pragma omp critical
global\_result += my\_result;

only one thread can execute the following structured block at a time



```
#include < stdio.h>
#include <stdlib.h>
#include <omp.h>
void Trap(double a, double b, int n, double* global_result_p);
int main(int argc, char* argv[]) {
   double global_result = 0.0; /* Store result in global_result */
   double a, b;
                                /* Left and right endpoints
                                                                  */
   int n;
                                 /* Total number of trapezoids
                                                                  */
   int thread count;
   thread_count = strtol(argv[1], NULL, 10);
   printf("Enter a, b, and n\n");
   scanf("%lf %lf %d", &a, &b, &n);
  pragma omp parallel num_threads(thread_count)
#
   Trap(a, b, n, &global_result);
   printf("With n = %d trapezoids, our estimate\n", n);
   printf("of the integral from %f to %f = %.14e\n",
      a, b, global result);
   return 0;
   /* main */
```



```
void Trap(double a, double b, int n, double* global_result_p) {
   double h, x, my result;
   double local a, local b;
   int i, local n;
   int my rank = omp_get_thread_num();
   int thread_count = omp_get_num_threads();
   h = (b-a)/n;
   local n = n/thread count;
   local_a = a + my_rank*local_n*h;
   local b = local a + local n*h;
   my result = (f(local a) + f(local b))/2.0;
   for (i = 1; i <= local_n-1; i++) {</pre>
     x = local a + i*h;
     my result += f(x);
   }
   my result = my result*h;
# pragma omp critical
   *global_result_p += my_result;
} /* Trap */
```





### **SCOPE OF VARIABLES**



In serial programming, the scope of a variable consists of those parts of a program in which the variable can be used.

In OpenMP, the scope of a variable refers to the set of threads that can access the variable in a parallel block.



#### **Scope in OpenMP**

A variable that can be accessed by all the threads in the team has shared scope.

A variable that can only be accessed by a single thread has private scope.

The default scope for variables declared before a parallel block is shared.







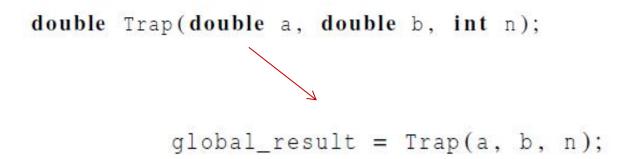
### THE REDUCTION CLAUSE



We need this more complex version to add each thread's local calculation to get *global\_result*.

void Trap(double a, double b, int n, double\* global\_result\_p);

#### Although we'd prefer this.





If we use this, there's no critical section!

```
double Local_trap(double a, double b, int n);
```

#### If we fix it like this...

```
global_result = 0.0;
# pragma omp parallel num_threads(thread_count)
{
# pragma omp critical
global_result += Local_trap(double a, double b, int n);
}
```

... we force the threads to execute sequentially.



We can avoid this problem by declaring a private variable inside the parallel block and moving the critical section after the function call.

```
global_result = 0.0;
# pragma omp parallel num_threads(thread_count)
{
    double my_result = 0.0; /* private */
    my_result += Local_trap(double a, double b, int n);
# pragma omp critical
    global_result += my_result;
}
```







#### **Reduction operators**

- A reduction operator is a binary operation (such as addition or multiplication).
- A reduction is a computation that repeatedly applies the same reduction operator to a sequence of operands in order to get a single result.
- All of the intermediate results of the operation should be stored in the same variable: the reduction variable.

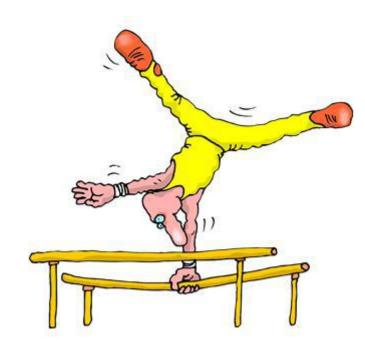


A reduction clause can be added to a parallel directive.

reduction(<operator>: <variable list>) +, \*, -, &, |, ^, &&, ||

global\_result = 0.0;
# pragma omp parallel num\_threads(thread\_count) \
 reduction(+: global\_result)
global\_result += Local\_trap(double a, double b, int n);





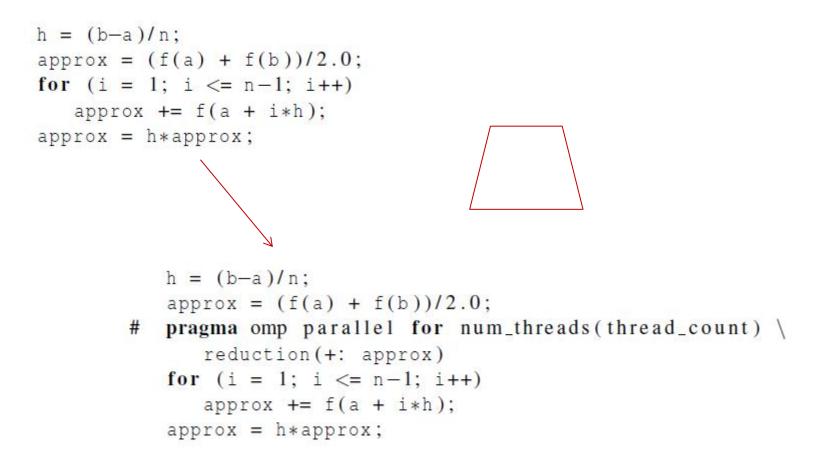
### THE "PARALLEL FOR" DIRECTIVE



#### **Parallel for**

- Forks a team of threads to execute the following structured block.
- However, the structured block following the parallel for directive must be a for loop.
- Furthermore, with the parallel for directive the system parallelizes the for loop by dividing the iterations of the loop among the threads.







# Legal forms for parallelizable for statements

for	index = start	;	<pre>index &lt; end index &lt;= end index &gt;= end index &gt; end</pre>	;	index index += incr index -= incr index = index + incr index = incr + index	
(					index = index - incr	/



## **Caveats**

- The variable index must have integer or pointer type (e.g., it can't be a float).
- The expressions start, end, and incr must have a compatible type. For example, if index is a pointer, then incr must have integer type.

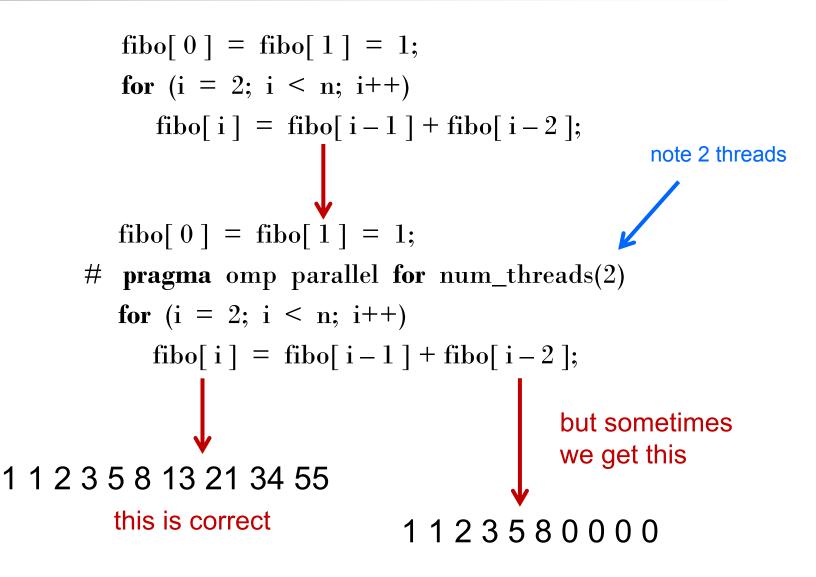


## Caveats

- The expressions start, end, and incr must not change during execution of the loop.
- During execution of the loop, the variable index can only be modified by the "increment expression" in the for statement.



## **Data dependencies**





## What happened?



- OpenMP compilers don't check for dependences among iterations in a loop that's being parallelized with a parallel for directive.
- 2. A loop in which the results of one or more iterations depend on other iterations cannot, in general, be correctly parallelized by OpenMP.



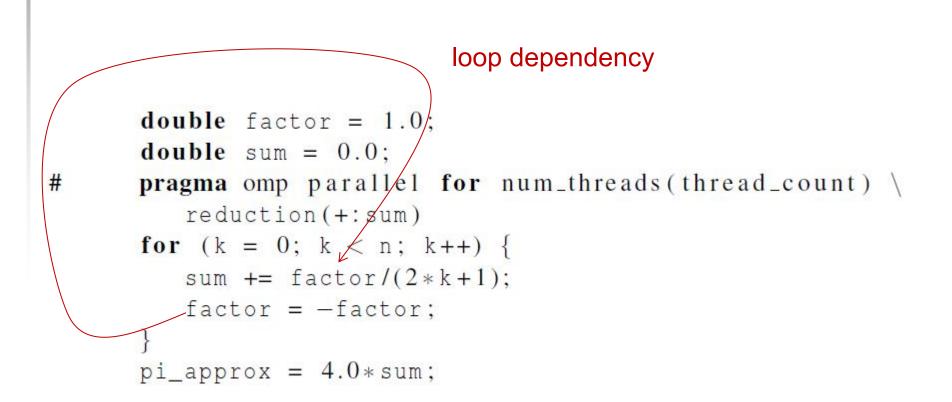
#### Estimating π

$$\pi = 4\left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right] = 4\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$$

double factor = 1.0; double sum = 0.0; for (k = 0; k < n; k++) { sum += factor/(2\*k+1); factor = -factor; } pi\_approx = 4.0\*sum;



## **OpenMP solution #1**





## **OpenMP solution #2**

```
double sum = 0.0;
pragma omp parallel for num_threads(thread_count) \
  reduction(+:sum) private(factor)
for (k = 0; k < n; k++) {
  if (k % 2 == 0)
    factor = 1.0;
  else
    factor = -1.0;
  sum += factor/(2*k+1);
}</pre>
```



#

## The default clause

Lets the programmer specify the scope of each variable in a block.

default(none)

With this clause the compiler will require that we specify the scope of each variable we use in the block and that has been declared outside the block.



## The default clause

```
double sum = 0.0;
pragma omp parallel for num_threads(thread_count) \
    default(none) reduction(+:sum) private(k, factor) \
    shared(n)
for (k = 0; k < n; k++) {
    if (k % 2 == 0)
        factor = 1.0;
    else
        factor = -1.0;
    sum += factor/(2*k+1);
}
```



#



# MORE ABOUT LOOPS IN OPENMP: SORTING



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## **Bubble Sort**

```
for (list_length = n; list_length >= 2; list_length--)
   for (i = 0; i < list_length - 1; i++)
      if (a[i] > a[i+1]) {
        tmp = a[i];
         a[i] = a[i+1];
         a[i+1] = tmp;
```



## **Serial Odd-Even Transposition Sort**

```
for (phase = 0; phase < n; phase++)
if (phase % 2 == 0)
for (i = 1; i < n; i += 2)
    if (a[i-1] > a[i]) Swap(&a[i-1],&a[i]);
else
for (i = 1; i < n-1; i += 2)
    if (a[i] > a[i+1]) Swap(&a[i], &a[i+1]);
```



## **Serial Odd-Even Transposition Sort**

	Subscript in Array			ray			
Phase	0		1		2	63167	3
0	9	$\leftrightarrow$	7		8	$\leftrightarrow$	6
	7		9		6		8
1	7		9	$\leftrightarrow$	6		8
	7		6		9		8
2	7	$\leftrightarrow$	6		9	$\leftrightarrow$	8
	6		7		8		9
3	6		7	$\leftrightarrow$	8		9
	6		7		8		9



#### First OpenMP Odd-Even Sort

```
for (phase = 0; phase < n; phase++) {
      if (phase \% 2 == 0)
#
         pragma omp parallel for num_threads(thread_count) \
            default(none) shared(a, n) private(i, tmp)
         for (i = 1; i < n; i += 2) {
            if (a[i-1] > a[i]) {
               tmp = a[i-1];
               a[i-1] = a[i];
               a[i] = tmp;
      else
#
         pragma omp parallel for num_threads(thread_count) \
            default(none) shared(a, n) private(i, tmp)
         for (i = 1; i < n-1; i += 2) {
            if (a[i] > a[i+1]) {
               tmp = a[i+1];
               a[i+1] = a[i];
               a[i] = tmp;
         }
   }
```



#### Second OpenMP Odd-Even Sort

```
pragma omp parallel num_threads(thread_count) \
#
      default(none) shared(a, n) private(i, tmp, phase)
   for (phase = 0; phase < n; phase++) {
      if (phase \% 2 == 0)
         pragma omp for
#
         for (i = 1; i < n; i += 2) {
            if (a[i-1] > a[i]) {
               tmp = a[i-1];
               a[i-1] = a[i];
               a[i] = tmp;
      else
#
         pragma omp for
         for (i = 1; i < n-1; i += 2) {
            if (a[i] > a[i+1]) {
               tmp = a[i+1];
               a[i+1] = a[i];
               a[i] = tmp;
         }
   }
```



Odd-even sort with two parallel for directives and two for directives. (Times are in seconds.)

thread_count	1	2	3	4
Two parallel for directives	0.770	0.453	0.358	0.305
Two for directives	0.732	0.376	0.294	0.239





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# **SCHEDULING LOOPS**



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We want to parallelize this loop.

sum = 0.0;
for (i = 0; i <= n; i++)
 sum += f(i);</pre>

Thread	Iterations		
0	$0, n/t, 2n/t, \ldots$		
1	$1, n/t+1, 2n/t+1, \ldots$		
	:		
<i>t</i> – 1	$t-1, n/t+t-1, 2n/t+t-1, \ldots$		

Assignment of work using cyclic partitioning.



```
double f(int i) {
    int j, start = i*(i+1)/2, finish = start + i;
    double return_val = 0.0;
    for (j = start; j <= finish; j++) {
        return_val += sin(j);
    }
    return return_val;
} /* f */</pre>
```

#### Our definition of function *f*.



## **Results**

- f(i) calls the sin function *i* times.
- Assume the time to execute f(2i) requires approximately twice as much time as the time to execute f(i).

- n = 10,000
  - one thread
  - run-time = 3.67 seconds.



## **Results**

#### n = 10,000

- two threads
- default assignment
- run-time = 2.76 seconds
- speedup = 1.33
- n = 10,000
  - two threads
  - cyclic assignment
  - run-time = 1.84 seconds
  - speedup = 1.99





## **The Schedule Clause**

```
Default schedule:
```

```
sum = 0.0;
```

pragma omp parallel for num\_threads(thread\_count) \
 reduction(+:sum)
for (i = 0; i <= n; i++)
 sum += f(i);</pre>

#### Cyclic schedule:

```
sum = 0.0;
pragma omp parallel for num_threads(thread_count) \
    reduction(+:sum) schedule(static,1)
for (i = 0; i <= n; i++)
    sum += f(i);</pre>
```



#

#

# schedule (type, chunksize)

#### Type can be:

- static: the iterations can be assigned to the threads before the loop is executed.
- dynamic or guided: the iterations are assigned to the threads while the loop is executing.
- auto: the compiler and/or the run-time system determine the schedule.
- runtime: the schedule is determined at runtime.

#### The chunksize is a positive integer.



## **The Static Schedule Type**

#### twelve iterations, 0, 1, ..., 11, and three threads

schedule(static,1)

Thread 0: 0,3,6,9 Thread 1: 1,4,7,10 Thread 2: 2,5,8,11



## **The Static Schedule Type**

#### twelve iterations, 0, 1, ..., 11, and three threads

schedule(static,2)

Thread 0: 0,1,6,7 Thread 1: 2,3,8,9 Thread 2: 4,5,10,11



## **The Static Schedule Type**

twelve iterations, 0, 1, ..., 11, and three threads

schedule(static,4)

Thread 0: 0,1,2,3 Thread 1: 4,5,6,7 Thread 2: 8,9,10,11



# The Dynamic Schedule Type

- The iterations are also broken up into chunks of chunksize consecutive iterations.
- Each thread executes a chunk, and when a thread finishes a chunk, it requests another one from the run-time system.
- This continues until all the iterations are completed.
- The chunksize can be omitted. When it is omitted, a chunksize of 1 is used.



## **The Guided Schedule Type**

- Each thread also executes a chunk, and when a thread finishes a chunk, it requests another one.
- However, in a guided schedule, as chunks are completed the size of the new chunks decreases.
- If no chunksize is specified, the size of the chunks decreases down to 1.
- If chunksize is specified, it decreases down to chunksize, with the exception that the very last chunk can be smaller than chunksize.



Thread	Chunk	Size of Chunk	<b>Remaining Iterations</b>
0	1 - 5000	5000	4999
1	5001 - 7500	2500	2499
1	7501 - 8750	1250	1249
1	8751 - 9375	625	624
0	9376 - 9687	312	312
1	9688 - 9843	156	156
0	9844 - 9921	78	78
1	9922 - 9960	39	39
1	9961 - 9980	20	19
1	9981 - 9990	10	9
1	9991 - 9995	5	4
0	9996 - 9997	2	2
1	9998 - 9998	1	1
0	9999 - 9999	1	0

Assignment of trapezoidal rule iterations 1–9999 using a guided schedule with two threads.



## **The Runtime Schedule Type**

- The system uses the environment variable OMP\_SCHEDULE to determine at runtime how to schedule the loop.
- The OMP\_SCHEDULE environment variable can take on any of the values that can be used for a static, dynamic, or guided schedule.





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## Queues

- Can be viewed as an abstraction of a line of customers waiting to pay for their groceries in a supermarket.
- A natural data structure to use in many multithreaded applications.
- For example, suppose we have several "producer" threads and several "consumer" threads.
  - Producer threads might "produce" requests for data.
  - Consumer threads might "consume" the request by finding or generating the requested data.



## **Message-Passing**

- Each thread could have a shared message queue, and when one thread wants to "send a message" to another thread, it could enqueue the message in the destination thread's queue.
- A thread could receive a message by dequeuing the message at the head of its message queue.



## **Message-Passing**

```
for (sent_msgs = 0; sent_msgs < send_max; sent_msgs++) {
    Send_msg();
    Try_receive();
}</pre>
```

```
while (!Done())
Try_receive();
```



## **Sending Messages**

mesg = random(); dest = random() % thread\_count; # pragma omp critical Enqueue(queue, dest, my\_rank, mesg);



#### **Receiving Messages**

if (queue\_size == 0) return; else if (queue\_size == 1) # pragma omp critical Dequeue(queue, &src, &mesg); else Dequeue(queue, &src, &mesg); Print\_message(src, mesg);



#### **Termination Detection**

queue\_size = enqueued - dequeued; if (queue\_size == 0 && done\_sending == thread\_count) return TRUE; else return FALSE;

each thread increments this after completing its for loop



# **Startup (1)**

- When the program begins execution, a single thread, the master thread, will get command line arguments and allocate an array of message queues: one for each thread.
- This array needs to be shared among the threads, since any thread can send to any other thread, and hence any thread can enqueue a message in any of the queues.



# **Startup (2)**

- One or more threads may finish allocating their queues before some other threads.
- We need an explicit barrier so that when a thread encounters the barrier, it blocks until all the threads in the team have reached the barrier.
- After all the threads have reached the barrier all the threads in the team can proceed.
   # pragma omp barrier



## **The Atomic Directive (1)**

 Unlike the critical directive, it can only protect critical sections that consist of a single C assignment statement.

# pragma omp atomic

Further, the statement must have one of the following forms:

x <op>= <expression>; x++; ++x; x--; --x;



## **The Atomic Directive (2)**

Here <op> can be one of the binary operators

+, \*, -, /, &, ^, |, <<, or >>

- Many processors provide a special loadmodify-store instruction.
- A critical section that only does a loadmodify-store can be protected much more efficiently by using this special instruction rather than the constructs that are used to protect more general critical sections.



### **Critical Sections**

 OpenMP provides the option of adding a name to a critical directive:

# pragma omp critical(name)

When we do this, two blocks protected with critical directives with different names can be executed simultaneously.

 However, the names are set during compilation, and we want a different critical section for each thread's queue.



#### Locks

A lock consists of a data structure and functions that allow the programmer to explicitly enforce mutual exclusion in a critical section.





#### Locks

/\* Executed by one thread \*/
Initialize the lock data structure;

/\* Executed by multiple threads \*/
Attempt to lock or set the lock data structure;
Critical section;
Unlock or unset the lock data structure;
. . .
/\* Executed by one thread \*/

Destroy the lock data structure;



## Using Locks in the Message-Passing Program

# pragma omp critical
/\* q\_p = msg\_queues[dest] \*/
Enqueue(q\_p, my\_rank, mesg);

/\* q\_p = msg\_queues[dest] \*/
omp\_set\_lock(&q\_p->lock);
Enqueue(q\_p, my\_rank, mesg);
omp\_unset\_lock(&q\_p->lock);



## Using Locks in the Message-Passing Program

# pragma omp critical
/\* q\_p = msg\_queues[my\_rank] \*/
Dequeue(q\_p, &src, &mesg);

/\* q\_p = msg\_queues[my\_rank] \*/
omp\_set\_lock(&q\_p->lock);
Dequeue(q\_p, &src, &mesg);
omp\_unset\_lock(&q\_p->lock);



## **Some Caveats**

- 1. You shouldn't mix the different types of mutual exclusion for a single critical section.
- 2. There is no guarantee of fairness in mutual exclusion constructs.
- 3. It can be dangerous to "nest" mutual exclusion constructs.



### **Matrix-vector multiplication**

<i>a</i> <sub>00</sub>	<i>a</i> <sub>01</sub>		$a_{0,n-1}$		УО
<i>a</i> <sub>10</sub>	<i>a</i> <sub>11</sub>		$a_{1,n-1}$	<i>x</i> <sub>0</sub>	<i>y</i> 1
:	:		:	<i>x</i> <sub>1</sub>	:
$a_{i0}$	$a_{i1}$	•••	$a_{i,n-1}$	:	$y_i = a_{i0}x_0 + a_{i1}x_1 + \cdots + a_{i,n-1}x_{n-1}$
:	:		:	$x_{n-1}$	:
$a_{m-1,0}$	$a_{m-1,1}$	•••	$a_{m-1,n-1}$		<u><i>Ym</i>-1</u>

$$y_i = a_{i0}x_0 + a_{i1}x_1 + \dots + a_{i,n-1}x_{n-1}$$



### **Matrix-vector multiplication**

# pragma omp parallel for num\_threads(thread\_count) \
 default(none) private(i, j) shared(A, x, y, m, n)
for (i = 0; i < m; i++) {
 y[i] = 0.0;
 for (j = 0; j < n; j++)
 y[i] += A[i][j]\*x[j];
 }
 Run-times and efficiencies
 of matrix-vector multiplication
 (times are in seconds)</pre>

	Matrix Dimension								
	8,000,	$000 \times 8$	$8000 \times 8000$		$8 \times 8,000,000$				
Threads	Time	Eff.	Time	Eff.	Time	Eff.			
1	0.322	1.000	0.264	1.000	0.333	1.000			
2	0.219	0.735	0.189	0.698	0.300	0.555			
4	0.141	0.571	0.119	0.555	0.303	0.275			



```
void Tokenize(
     char* lines[] /* in/out */, Thread-Safety
     int line_count /* in */,
     int thread_count /* in */) {
   int my rank, i, j;
   char *my token;
#
  pragma omp parallel num_threads(thread_count) \
      default(none) private(my_rank, i, j, my_token) \
     shared(lines, line count)
   {
     my rank = omp get thread num();
#
     pragma omp for schedule (static, 1)
     for (i = 0; i < line count; i++) {
        printf("Thread %d > line %d = %s", my rank, i, lines[i]);
        i = 0:
        my token = strtok(lines[i], " \t\n");
        while ( my token != NULL ) {
           printf("Thread %d > token %d = %s\n", my_rank, j, my_token);
           my token = strtok(NULL, " \t\n");
           j++;
      } /* for i */
     /* omp parallel */
```



# **Concluding Remarks (1)**

- OpenMP is a standard for programming shared-memory systems.
- OpenMP uses both special functions and preprocessor directives called pragmas.
- OpenMP programs start multiple threads rather than multiple processes.
- Many OpenMP directives can be modified by clauses.



# **Concluding Remarks (2)**

- A major problem in the development of shared memory programs is the possibility of race conditions.
- OpenMP provides several mechanisms for insuring mutual exclusion in critical sections.
  - Critical directives
  - Named critical directives
  - Atomic directives
  - Simple locks



# **Concluding Remarks (3)**

- By default most systems use a blockpartitioning of the iterations in a parallelized for loop.
- OpenMP offers a variety of scheduling options.
- In OpenMP the scope of a variable is the collection of threads to which the variable is accessible.



## **Concluding Remarks (4)**

A reduction is a computation that repeatedly applies the same reduction operator to a sequence of operands in order to get a single result.

