EXERCÍCIOS

(1) Para cada uma das fórmulas abaixo determine se é um axioma lógico.

(a) $\forall x \forall z (x \doteq y \to (x \doteq c \to x \doteq y))$

(b) $x \doteq y \to (y \doteq z \to z \doteq x)$

(c) $\forall z (x \doteq y \to (x \doteq c \to y \doteq c))$

(d) $\forall w \exists x (P(w,x) \to P(w,w)) \to \exists x (P(x,x) \to P(x,x))$

(e) $\forall x (\forall x (c \doteq f(x,c) \to \forall x \forall x (c \doteq f(x,c))))$

(f) $(\exists x P(x) \to \exists y \forall z R(z, f(y))) \to ((\exists x P(x) \to \forall y \neg \forall z R(z, f(y))) \to \forall x \neg P(x))$

(g) $\forall x \exists y (\neg(y \doteq x)) \to \exists y (\neg(y \doteq 0))$

(h) $\forall x \exists y (\neg(x \doteq y))$

(i) $\forall x \exists y (y < x) \to \exists y (y < y)$

(j) $\forall x ((0 < x) \to (0 < x + x)) \to ((0 < x) \to \forall x (0 < x + x))$

(k) $\forall x \forall y ((x + 1 \doteq 0) \to ((y \doteq 1) \to (x + 1 \doteq 0)))$

(2) Escreva uma prova para

(a) $\vdash (\forall x_1 (R_1^1(x_1) \vee R_2^1(x_1))) \to (R_2^1(x_1) \vee R_1^1(x_1))$.

(b) $\vdash R_1^1(c_1) \to (R_1^1(c_2) \to (R_1^1(c_1) \wedge r_1^1(c_2)))$.

(c) $\vdash \forall x_1 (\forall x_2 (R_1^1(x_1) \to (R_1^1(x_2) \to R_1^1(x_1))))$.

(d) $\{Q(x), \forall y (Q(y) \to \forall z P(z))\} \vdash \forall x P(x)$

(e) $\vdash (\alpha \to \exists x \beta) \leftrightarrow \exists x (\alpha \to \beta)$ se x não ocorre livre em α

(f) $\vdash (\forall x \beta \to \alpha) \leftrightarrow \exists x (\beta \to \alpha)$ se x não ocorre livre em α

(g) $\vdash P(y) \leftrightarrow \forall x (x \doteq y \to P(x))$

(3) Demonstre as propriedades de $\vdash$.