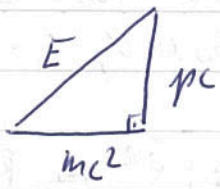
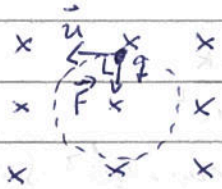
$$E^2 = p^2c^2 + (mc^2)^2$$



Caso extremamente relativístico (ultrarrelativístico)

para $E >> mc^2 \Rightarrow E \simeq pc$ (< 1% para $E > 8mc^2$)
(ou para $m=0 \Rightarrow E = pc$)

como $p \simeq E/c \Rightarrow [p] = MeV/c$



$\vec{F} = q\vec{u}\times\vec{B} \Rightarrow \vec{F} \perp \vec{B}$, $\vec{F} \perp \vec{u}$ $\Rightarrow$

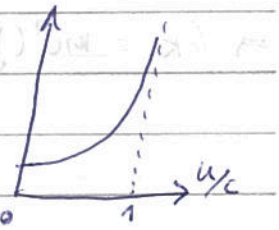
$\Rightarrow W^F = 0 \Rightarrow \frac{dE}{dt} = 0 \Rightarrow E = const. \Rightarrow p = const.$

$$\vec{F} = q\vec{u}\times\vec{B} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m_p\vec{u}) = m_p\frac{d\vec{u}}{dt}$$

no caso $\vec{u} \perp \vec{B} \Rightarrow$ mov. circ. uniforme com $a_c = u^2/R \Rightarrow$

$\Rightarrow quB = m_p\left|\frac{du}{dt}\right| = m_p\left(\frac{u^2}{R}\right) \Rightarrow$

$\Rightarrow qBR = m_p u = p \quad : p = qBR/mu$



$$p = qBR\cdot\frac{e}{e} = \left(\frac{q}{e}\right)BR\cdot e$$

$\Rightarrow [p] = kg\,m/s = 1{,}87\cdot10^{21}\ MeV/c \cdot 1{,}6\cdot10^{-19}\ C \simeq 300$

$\Rightarrow \boxed{p = 300\left(\frac{q}{e}\right)BR}$, onde $[p] = MeV/c$, $[B] = T$, $[R] = m$, $[q] = C$

/ /

Ex. 2-14) elétron num campo magnético

$E = 30\ MeV$, $B = 500\ G = 0{,}05\ T$, $R = ?$

$$p = 300 \left(\frac{q}{e}\right) BR \Rightarrow R = \frac{p}{300B}$$

como $E = 300\ MeV \gg mc^2 = 0{,}511\ MeV \Rightarrow p \simeq E/c = 30\ MeV/c$

$$\Rightarrow R = \frac{30\ MeV/c}{300 \cdot 0{,}05} = 2\ m$$

Caso não-relativístico : $\gamma \to 1$

$$E = m\gamma c^2 \Rightarrow \gamma = \frac{E}{mc^2} = \frac{mc^2 + E_K}{mc^2} = 1 + \frac{E_K}{mc^2}$$

para $E_K \ll mc^2 \Rightarrow \gamma \simeq 1$ (aproximação não-relativística)

$$E_K = mc^2(\gamma - 1) = mc^2\left[(1 - u^2/c^2)^{-1/2} - 1\right] \simeq mc^2\left(1 + \frac{1}{2}\frac{u^2}{c^2} + \dots - 1\right) \simeq \frac{1}{2}mu^2$$

$$\Rightarrow E_K \simeq \frac{1}{2}mu^2 = \frac{p^2}{2m}$$

expandindo mais 1 termo : $\gamma \simeq \left(1 - u^2/c^2\right)^{-1/2} \simeq 1 + \frac{u^2}{2c^2} + \frac{3}{8}\frac{u^4}{c^4} + \dots$

$$\Rightarrow E_K = mc^2(\gamma - 1) \simeq \frac{1}{2}mu^2 + mc^2\,\frac{3}{2}\left(\frac{u^4}{4c^4}\right)\cdot\frac{m}{m} + \dots$$

$$\simeq \frac{1}{2}mu^2 + \frac{3}{2}\frac{\left(\frac{1}{2}mu^2\right)^2}{mc^2} + \dots$$

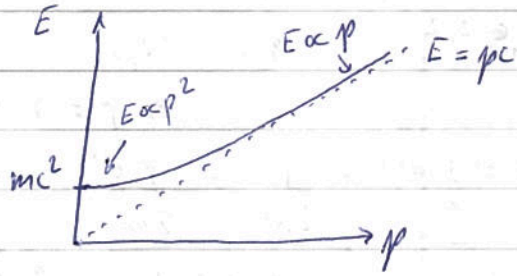
$$\Rightarrow E_K - \tfrac{1}{2}mu^2 \simeq \frac{3}{2}\frac{E_K^2}{mc^2} \Rightarrow \frac{E_K - \frac{1}{2}mu^2}{E_K} \simeq \frac{3}{2}\frac{E_K}{mc^2}$$

por exemplo, se $E_K/mc^2 \simeq 1\% \Rightarrow \dfrac{E_K - \frac{1}{2}mu^2}{E_K} \simeq 1{,}5\%$

Probl. 2-28) E ↑ p →

(2-36) $E \simeq pc$ $\quad (E \gg mc^2)$

(2-39) $E = 1 + \frac{E_K}{mc^2} \simeq \frac{p^2}{2m}$ $\quad (E \ll mc^2)$



$E = \sqrt{p^2c^2 + m^2c^4}$

quando $\gamma \gg 1$: $\gamma = \frac{1}{\sqrt{1-u^2/c^2}} = \frac{c}{\sqrt{(c^2-u^2)}} = \frac{c}{\sqrt{(c-u)(c+u)}} =$

$= \frac{c}{c\sqrt{(1-u/c)(1+u/c)}} \simeq \frac{1}{\sqrt{(1-u/c)2}} \Rightarrow$

$\Rightarrow 2\gamma^2 \simeq \frac{1}{1-u/c} \Rightarrow \boxed{\frac{u}{c} \simeq 1 - \frac{1}{2\gamma^2}}$ para $\gamma \gg 1$

ou a expressão exata: $\frac{u}{c} = \frac{m\gamma u c}{m\gamma c^2} = \frac{pc}{E}$

Ex. 2-15) partículas diferentes de mesma energia

$V = 10 \cdot 10^6\ V = 10\ MV$ $\quad (p$ ou $e)$

$E_K = qV = 10\ MeV \gg m_e c^2 = 0{,}511\ MeV$

$\ll m_p c^2 = 938{,}3\ MeV$

1.) elétron: $\gamma = 1 + \frac{E_K}{mc^2} = 1 + \frac{10}{0{,}511} = 20{,}57$

$E = mc^2 + E_K = 10{,}511\ MeV$

$\Rightarrow pc = \sqrt{E^2 - (mc^2)^2} = \sqrt{10{,}511^2 - 0{,}511^2} = 10{,}50\ MeV$

$\Rightarrow p = 10{,}50\ MeV/c$

na aprox. ultrarrelat.: $p \simeq E/c = 10{,}51\ MeV/c$

(2.40) $\frac{u}{c} \simeq 1 - \frac{1}{2\gamma^2} = 0{,}99882$

(2.10) $\frac{u}{c} = \frac{pc}{E} = 0{,}99905$

/ /

2.) protão: $\gamma = 1 + \frac{E_K}{mc^2} = 1 + \frac{10}{938,3} = 1,0107$

$E = mc^2 + E_K = 948,3 \text{ MeV}$

$\Rightarrow pc = \sqrt{E^2 - (mc^2)^2} = \sqrt{948,3^2 - 938,3^2} = 137,35 \text{ MeV}$

$\Rightarrow p = 137,35 \text{ MeV/c}$

na aprox. não-relat.: $E_K \simeq \frac{1}{2} m u^2 \simeq \frac{p^2}{2m} \simeq \frac{p^2 c^2}{2mc^2} \Rightarrow$

$\Rightarrow pc \simeq \sqrt{2mc^2 E_K} = \sqrt{2 \cdot 938,3 \cdot 10} = 136,989 \text{ MeV}$

$\Rightarrow p = 136,989 \text{ MeV/c}$

$\frac{u}{c} = \frac{mu}{mc} \simeq \frac{p}{mc} = \frac{pc}{mc^2} = 0,145997$

ou $\frac{u}{c} = \frac{pc}{E} = \frac{137,35}{948,3} = 0,144842$