

# Fenômenos Mecânicos Laboratório: Fórmulas

## Medidas múltiplas da mesma grandeza

Valor médio dos valores  $x_1$  a  $x_N$ :  $\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i$

Desvio padrão da média:  $\sigma_{\bar{x}} = \sqrt{\frac{1}{N(N-1)} \sum_{i=1}^N (x_i - \bar{x})^2}$

## Propagação de erros

Caso geral ( $F = F(x_1, x_2, x_3, \dots)$ ):  $\sigma_F = \sqrt{\left(\frac{\partial F}{\partial x_1}\right)^2 (\sigma_{x_1})^2 + \left(\frac{\partial F}{\partial x_2}\right)^2 (\sigma_{x_2})^2 + \dots}$

Casos específicos ( $w = w(x, y, \dots)$ ):

$$w = x \pm y: \sigma_w^2 = \sigma_x^2 + \sigma_y^2$$

$$w = axy: \left(\frac{\sigma_w}{w}\right)^2 = \left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2$$

$$w = a(y/x): \left(\frac{\sigma_w}{w}\right)^2 = \left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2$$

$$w = x^m: \left|\frac{\sigma_w}{w}\right| = |m| \frac{\sigma_x}{x}$$

$$w = ax: \left|\frac{\sigma_w}{w}\right| = \left|\frac{\sigma_x}{x}\right| \text{ ou } \sigma_w = |a| \sigma_x$$

$$w = ax + b: \left|\frac{\sigma_w}{w}\right| = \left|\frac{\sigma_x}{x}\right| \text{ ou } \sigma_w = |a| \sigma_x$$

$$w = ax^p y^q: \left(\frac{\sigma_w}{w}\right)^2 = \left(p \frac{\sigma_x}{x}\right)^2 + \left(q \frac{\sigma_y}{y}\right)^2$$

$$w = a \sin(bx): \sigma_w = |ab \cos(bx)| \sigma_x \quad b\sigma_x \text{ em radianos}$$

## Reta de melhor ajuste

Valores medidos de  $X$ :  $x_i$ , de  $Y$ :  $y_i$ , erros experimentais de  $y_i$ :  $\sigma_i$

Coefficientes angular e linear da reta de melhor ajuste:  $a$  e  $b$

Residual (“chi-quadrado”):  $\chi^2(a, b) = \sum_i \left[ \frac{y_i - (ax_i + b)}{\sigma_i} \right]^2$

Método de Mínimos Quadrados (MMQ)

$$\langle \sigma^2 \rangle := \sum_i \frac{1}{\sigma_i^2}; \quad \langle x \rangle := \frac{1}{\langle \sigma^2 \rangle} \sum_i \frac{x_i}{\sigma_i^2}; \quad \langle x^2 \rangle := \frac{1}{\langle \sigma^2 \rangle} \sum_i \frac{x_i^2}{\sigma_i^2};$$

$$\langle y \rangle := \frac{1}{\langle \sigma^2 \rangle} \sum_i \frac{y_i}{\sigma_i^2}; \quad \langle xy \rangle := \frac{1}{\langle \sigma^2 \rangle} \sum_i \frac{x_i y_i}{\sigma_i^2}$$

$$a = \frac{\langle x \rangle \langle y \rangle - \langle xy \rangle}{\langle x \rangle^2 - \langle x^2 \rangle}$$

$$b = \langle y \rangle - a \langle x \rangle$$

$$\Delta a = \sqrt{\frac{1/\langle \sigma^2 \rangle}{\langle x \rangle^2 - \langle x^2 \rangle}}$$

$$\Delta b = \sqrt{\frac{\langle x^2 \rangle / \langle \sigma^2 \rangle}{\langle x \rangle^2 - \langle x^2 \rangle}}$$