1

Durante a colisão a única força externa relevante que atua no carro é a força do parapeito ($\vec{F}$)

$$\sum \vec{F}_{ex} = \vec{F} = \frac{d\vec{P}}{dt}$$

1 dimensão → x

$$F = \frac{dP}{dt}$$

$$\overline{F} = \frac{\Delta P}{\Delta t} = \frac{\cancel{P_f} - P_i}{t - t_0} = \frac{m(\overset{0}{\cancel{v_f}} - v_i)}{t - 0} = -\frac{mv}{t}$$

$$\overline{F} = \frac{-(2300kg)(15m/s)}{(0,54s)} = -63888,88 \cong -6,4 \times 10^4 N$$

2

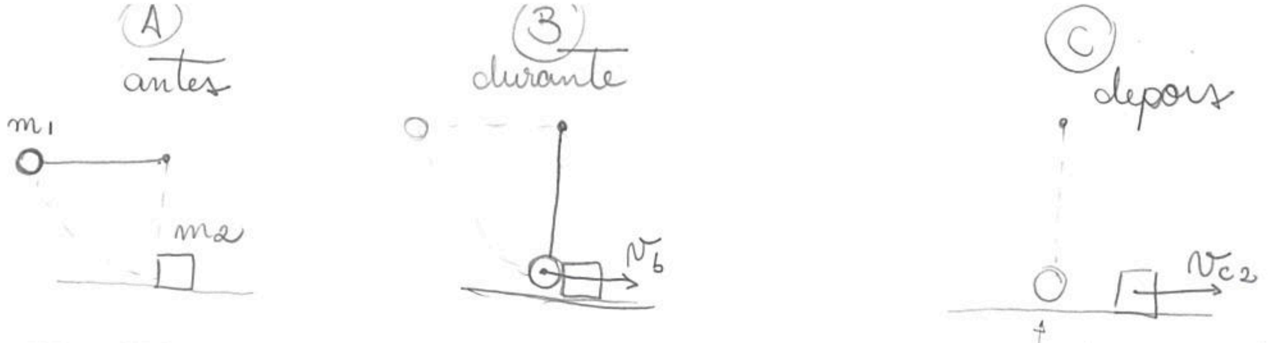
(a)

$$\overline{F} = \frac{\Delta p}{\Delta t} = \frac{p_f - p_i}{\Delta t} = \frac{m\,(v_f - v_i)}{\Delta t} = \frac{m\,(-v - v)}{\Delta t} = -\frac{2mv}{\Delta t}$$

(b)

$$\overline{F} = \frac{2\,(0{,}140\,kg)\,(7{,}8\,m/s)}{(3{,}9 \cdot 10^{-3}\,s)} = 560N$$

(3)



para onde vai a bola?
$\xleftarrow{v_{c1}}$ ou $\xrightarrow{v_{c1}}$?

Obs[1]:
vou considera $\xrightarrow{v_{c1}}$, caso encontre $v_{c1} < 0$
$\Rightarrow$ que o correto é $\xleftarrow{v_{c1}}$

O movimento de A até B $\rightarrow$ ausência de forças dissipativas
$\Downarrow$
conservação da energia mecânica

$$E_A = E_B$$

$$K_A + U_A = K_B + U_B$$

$$0 + m_1 g l = \frac{1}{2} m_1 v_b^2 + 0 \quad \Rightarrow \quad v_b = \sqrt{2gl} \cong 3{,}67 \text{ m/s}$$

(a) Durante o choque o momento linear é conservado $\Rightarrow B \rightarrow C$

$$\Sigma \vec{P}_B = \Sigma \vec{P}_C$$

$$m_1 v_b + m_2 (0) = m_1 v_{c1} + m_2 v_{c2}$$

$$m_1 (v_b - v_{c1}) = m_2 v_{c2} \quad [Eq\,1]$$

de B até C $\Rightarrow$ conservação da energia mecânica

$$\frac{1}{2} m_1 v_b^2 = \frac{1}{2} m_1 v_{c1}^2 + \frac{1}{2} m_2 v_{c2}^2$$

$$m_1 (v_b^2 - v_{c1}^2) = m_2 v_{c2}^2 \quad [Eq.2]$$

Temos o sistema:

$$\begin{cases} m_1 (v_b - v_{c_1}) = m_2 v_{c_2} \\ m_1 (v_b^2 - v_{c_1}^2) = m_2 v_{c_2}^2 \end{cases}$$

$$= \begin{cases} m_1^2 (v_b - v_{c_1})^2 = m_2^2 v_{c_2}^2 \\ m_1 (v_b^2 - v_{c_1}^2) = m_2 v_{c_2}^2 \end{cases}$$

$$\Rightarrow \cancel{m_1} (v_b^2 - v_{c_1}^2) = \cancel{m_2} \frac{m_1^{\cancel{2}}}{m_2^{\cancel{2}}} (v_b - v_{c_1})^2$$

$$m_2 (v_b^2 - v_{c_1}^2) = m_1 (v_b - v_{c_1})^2$$

Obs [2]:

$$(a-b)^2 = (a-b)(a-b)$$

$$(a^2 - b^2) = (a+b)(a-b)$$

$$\Rightarrow m_2 (v_b + v_{c_1})\cancel{(v_b - v_{c_1})} = m_1 (v_b - v_{c_1})\cancel{(v_b - v_{c_1})}$$

$$m_2 v_b + m_2 v_{c_1} = m_1 v_b - m_1 v_{c_1}$$

$$(m_1 + m_2) v_{c_1} = (m_1 - m_2) v_b$$

$$v_{c_1} = \frac{m_1 - m_2}{m_1 + m_2} v_b \Rightarrow v_{c_1} = -2{,}47 \, m/s$$

Obs [3]:

note que se $\begin{cases} m_1 > m_2 \Rightarrow v_{c_1} > 0 & \text{(continua no mesmo sentido)} \\ m_1 < m_2 \Rightarrow v_{c_1} < 0 & \text{(a bolinha volta)} \end{cases}$

(b) $m_1(v_b - v_{c1}) = m_2 v_{c2}$

$$v_{c2} = \frac{m_1}{m_2}\left[v_b - \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_b\right]$$

$$v_{c2} = \frac{m_1}{m_2} v_b \left[1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\right]$$

$$v_{c2} = \frac{m_1}{m_2} v_b \left[\frac{\cancel{m_1} + m_2 - \cancel{m_1} + m_2}{m_1 + m_2}\right]$$

$$v_{c2} = \frac{m_1}{\cancel{m_2}} v_b \left[\frac{2\cancel{m_2}}{m_1 + m_2}\right]$$

$$v_{c2} = \frac{2 m_1}{m_1 + m_2} v_b$$

$$v_{c2} = \frac{2\,(0{,}514)}{(0{,}514 + 2{,}63)}(3{,}67) \simeq 1{,}20 \text{ m/s}$$

(c) Agora: $K_C = \frac{1}{2} K_B$

$$\frac{1}{\cancel{2}} m_1 v_{c1}^2 + \frac{1}{\cancel{2}} m_2 v_{c2}^2 = \frac{1}{\cancel{2}}\left[\frac{1}{2} m_1 v_b^2 + \frac{1}{2} \cancel{m_2 (0)^2}\right]$$

$$m_1 v_{c1}^2 + m_2 v_{c2}^2 = \frac{1}{2} m_1 v_b^2$$

Pela conservação do momento de B → C

$$m_1 v_b = m_1 v_{c1} + m_2 v_{c2} \Rightarrow v_{c2} = \frac{m_1}{m_2}(v_b - v_{c1})$$

então $m_2 v_{c2}^2 = \frac{1}{2} m_1 v_b^2 + m_1 v_{c1}^2$

$$\cancel{m_2} \frac{m_1^2}{m_2^{\cancel{2}}}(v_b - v_{c1})^2 = m_1\left(\frac{1}{2} v_b^2 - v_{c1}^2\right)$$

$$m_1 (v_b - v_{c1})^2 = m_2\left(\frac{1}{2} v_b^2 - v_{c1}^2\right)$$

$$\frac{1}{2} m_2 v_b^2 - m_2 v_{c1}^2 = m_1 \left(v_b - v_{c1} \right)^2$$

$$m_2 v_b^2 - 2 m_2 v_{c1}^2 = 2 m_1 \left(v_b^2 - 2 v_b v_{c1} - v_{c1}^2 \right)$$

$$2 (m_1 + m_2) v_{c1}^2 - 4 m_1 v_b v_{c1} + (2 m_1 - m_2) v_b^2 = 0$$

$a = 2(m_1 + m_2)$

$b = -4 m_1 v_b$

$c = (2m_1 - m_2) v_b^2$

$\Rightarrow a v_{c1}^2 + b v_{c1} + c = 0$

$$v_{c1} = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$\Delta = b^2 - 4ac$

então ! $\Delta = 16 m_1^2 v_b^2 - 4 \left[2(m_1 + m_2) \right] \left[(2m_1 - m_2) v_b^2 \right]$

$$\Delta = 4 v_b^2 \left[\cancel{4 m_1^2} - \cancel{4 m_1^2} + 2 m_1 m_2 - 4 m_2 m_1 + 2 m_2^2 \right]$$

$$\Delta = 4 v_b^2 \left[2 m_2^2 - 2 m_1 m_2 \right] = 4 v_b^2 \left[2 m_2 (m_2 - m_1) \right]$$

$$v_{c1} = \frac{+4 m_1 v_b \pm 2 v_b \sqrt{2 m_2 (m_2 - m_1)}}{2 \cdot 2 (m_1 + m_2)}$$

$$v_{c1} = \frac{2 m_1 v_b \pm v_b \sqrt{2 m_2 (m_2 - m_1)}}{2 (m_1 + m_2)}$$

duas possibilidades :

$$\begin{cases} v_{c1} = -1{,}35 \, m/s \\ v_{c1} = 2{,}55 \, m/s \end{cases}$$

mas como $m_1 < m_2 \Rightarrow v_{c1} < 0$

portanto $v_{c1} = -1{,}35 \, m/s$

Como $m_1(v_b - v_{c1}) = m_2 v_{c2}$

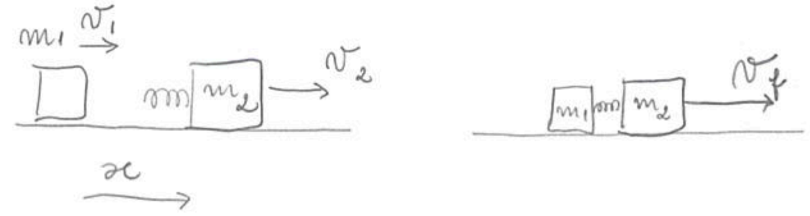
$$v_{c2} = \frac{m_1(v_b - v_{c1})}{m_2}$$

$$v_{c2} = \frac{(0{,}514\,kg)(3{,}67 - (-1{,}35))}{(2{,}63\,kg)}$$

$$v_{c2} = \frac{(0{,}514)(5{,}02)}{(2{,}63)}$$

$$v_{c2} = 0{,}98\,m/s$$

(4) antes depois



$k = 11,2 N/cm$
$= 11,2 . 10^{-2} N/m$

conservação do momento

$P_A = P_B$

$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f$

$$v_f = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

não há forças dissipativas $\Rightarrow$ conservação da energia mecânica

$E_A = E_B$

$K_{1A} + K_{2A} = K_{1B} + K_{2B} + U$

$$\frac{1}{\cancel{2}} m_1 v_1^2 + \frac{1}{\cancel{2}} m_2 v_2^2 = \frac{1}{\cancel{2}} (m_1 + m_2) v_f^2 + \frac{1}{2} k \Delta x^2$$

$$k \Delta x^2 = m_1 v_1^2 + m_2 v_2^2 - \cancel{(m_1 + m_2)} \frac{(m_1 v_1 + m_2 v_2)^2}{(m_1 + m_2)^{\cancel{2}}}$$

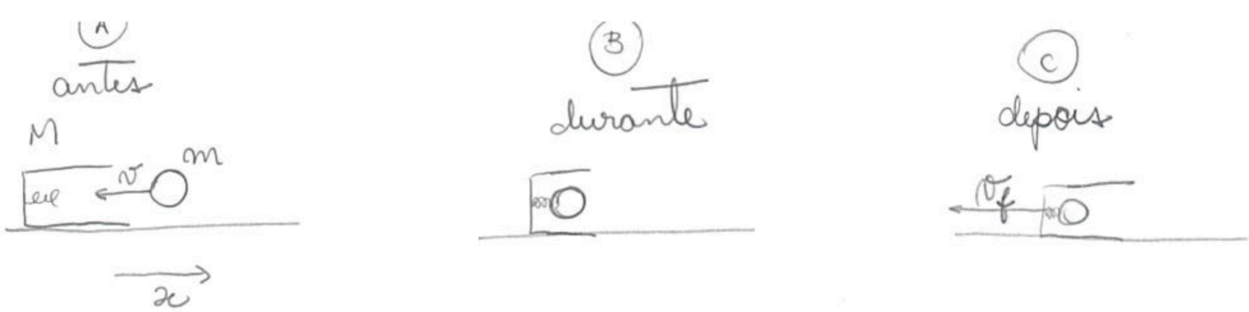
$$k \Delta x^2 = \frac{\cancel{m_1^2 v_1^2} + m_1 m_2 v_1^2 + m_1 m_2 v_2^2 + \cancel{m_2^2 v_2^2} - \cancel{m_1^2 v_1^2} - 2 m_1 m_2 v_1 v_2 - \cancel{m_2^2 v_2^2}}{(m_1 + m_2)}$$

$$k \Delta x^2 = \frac{m_1 m_2 (v_1^2 + v_2^2 - 2 v_1 v_2)}{m_1 + m_2} = \frac{m_1 m_2 (v_1 - v_2)^2}{m_1 + m_2}$$

$$\Delta x = (v_1 - v_2) \sqrt{\frac{m_1 m_2}{k (m_1 + m_2)}}$$

$$\Delta x = (10,3 - 3,27) \sqrt{\frac{(1,88)(4,92)}{(11,2.10^{-2})(1,88 + 4,92)}} = 0,245 m = 24,5 cm$$

5



a $P_A = P_C$

$$m v = (m+M)\, v_f$$

$$\boxed{v_f = \frac{m}{(m+M)}\, v}$$

b $U_m = K_i - K_f$

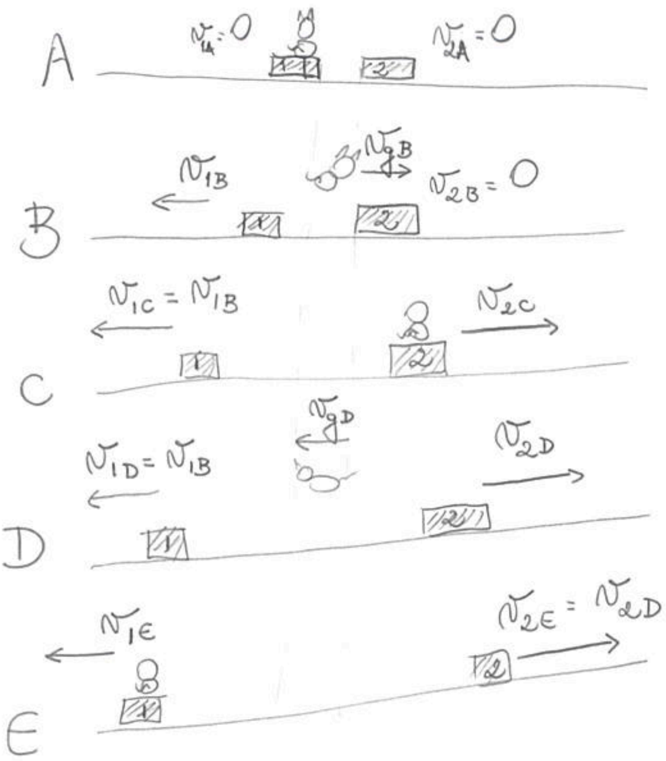
$$f = \frac{U_m}{K_i} = \frac{K_i - K_f}{K_i}$$

$$f = \frac{\frac{1}{2} m v^2 - \frac{1}{2}(m+M) v_f^2}{\frac{1}{2} m v^2} = \frac{\cancel{\frac{1}{2}}\left[m v^2 - (m+M) v_f^2\right]}{\cancel{\frac{1}{2}}\, m v^2} = \frac{m v^2 - (m+M) v_f^2}{m v^2}$$

$$f = \frac{m v^2 - (m+M)\left[\frac{m^2}{(m+M)^2} v^2\right]}{m v^2} = \frac{\cancel{m v^2}\left[1 - \frac{m}{(m+M)}\right]}{\cancel{m v^2}} = 1 - \frac{m}{m+M}$$

$$f = \frac{\cancel{m} + M - \cancel{m}}{m+M} \Rightarrow \boxed{f = \frac{M}{m+M}}$$

5) Esquema do problema:



$v_{gA} = 0$

$v_{gB} = v_{1B} + v_{gt}$

$v_{gC} = v_{2C}$

$v_{gD} = v_{2D} + v_{gt}$

$v_{gE} = v_{1E}$

A velocidade o salto do gato (v_{gt}) é em relação ao trenó ⇒ temos que passar para o referencial do solo

⇒ A velocidade do gato em relação ao solo (v_g) fica:

$$v_g = v_t + v_{gt}$$

↳ velocidade do trenó

momento linear de $A \to B$

$$P_A = P_B$$

$$(m+M)\cancel{v_{1A}}^{\,0} + M\cancel{v_{2A}}^{\,0} = Mv_{1B} + m\,v_{gB} + M\cancel{v_{2B}}^{\,0}$$

$$0 = Mv_{1B} + m\left(v_{1B} + v_{gt}\right)$$

$$(M+m)\,v_{1B} = -\,m\,v_{gt}$$

$$\boxed{v_{1B} = \frac{-\,m}{(m+M)}\,v_{gt}}$$

momento linear $B \rightarrow C$

$$\sum P_B = \sum P_C$$

$$P_{gB} + P_{1B} + P_{2B} = P_{gC} + P_{1C} + P_{2C}$$

$$m\, v_{gB} + \cancel{M}\, v_{1B} + M\, \cancel{v_{2B}}^{\,0} = m\, v_{gC} + \cancel{M}\, v_{1C} + M\, v_{2C}$$

mas: $v_{1C} = v_{1B}$

$$\Rightarrow m\, v_{gB} = m\, v_{gC} + M\, v_{2C}$$

mas $\begin{cases} v_{gB} = v_{1B} + v_{g\tau} \\ v_{gC} = v_{2C} \end{cases}$

$$\therefore m\, v_{1B} + m\, v_{g\tau} = (m+M)\, v_{2C}$$

mas $v_{1B} = \dfrac{-m}{(m+M)}\, v_{g\tau}$

$$(m+M)\, v_{2C} = -\frac{m^2}{(m+M)}\, v_{g\tau} + m\, v_{g\tau}$$

$$(m+M)\, v_{2C} = \left[\frac{-\cancel{m^2} + \cancel{m^2} + mM}{(m+M)}\right] v_{g\tau}$$

$$v_{2C} = \frac{mM}{(m+M)^2}\, v_{g\tau}$$

momento linear $C \to D$

$$\sum P_C = \sum P_D$$

$$P_{1C} + P_{gC} + P_{2C} = P_{1D} + P_{gD} + P_{2D}$$

$$M v_{1C} + m v_{gC} + M v_{2C} = M v_{1D} + m v_{gD} + M v_{2D}$$

$$\begin{cases} v_{gC} = v_{2C} \\ v_{gD} = v_{2D} + v_{gt} \\ v_{1D} = v_{1C} = v_{1B} \end{cases}$$

$$\cancel{M v_{1B}} + (m + M) v_{2C} = \cancel{M v_{1B}} + m\left(v_{2D} + v_{gt}\right) + M v_{2D}$$

$$(m + M) v_{2C} = m v_{gt} + (m + M) v_{2D}$$

$$(m + M) v_{2D} = m v_{gt} - (m + M) v_{2C}$$

$$(m + M) v_{2D} = m v_{gt} - \cancel{(m + M)} \left[\frac{m M}{(m + M)^{\cancel{2}}} v_{gt} \right]$$

$$(m + M) v_{2D} = \left[m - \frac{m M}{(m + M)} \right] v_{gt}$$

$$(m + M) v_{2D} = \frac{(m^2 + \cancel{mM} - \cancel{mM})}{(m + M)} v_{gt}$$

$$v_{2D} = \frac{m^2}{(m + M)^2} v_{gt}$$

e finalmente $A \to E$

$$\sum P_A = \sum P_E$$

$$0 = (m+M)\, v_{1E} + M\, v_{2E}$$

mas: $v_{2E} = v_{2D} = \dfrac{m^2}{(m+M)^2}\, v_{gT}$

$$(m+M)\, v_{1E} = -\frac{M m^2}{(m+M)^2}\, v_{gT}$$

$$v_{1E} = \frac{-M m^2}{(m+M)^3}\, v_{gT}$$

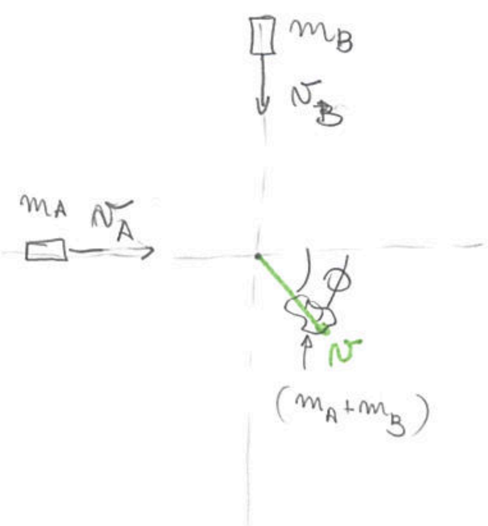
substituindo os valores

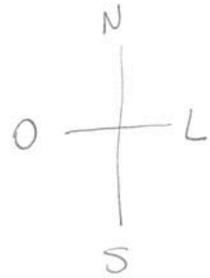
$$v_{1E} = \frac{-(22{,}7\,kg)(3{,}63\,kg)^2}{(22{,}7+3{,}63)^3}\,(3{,}05\,m/s) = -0{,}0499 \Rightarrow v_{1E} \simeq -50.10^{-3}\,m/s$$

$$v_{2E} = \frac{m^2}{(m+M)^2}\, v_{gT}$$

$$v_{2E} = \frac{(3{,}63)^2}{(22{,}7+3{,}63)^2}\,(3{,}05) = 0{,}058 \Rightarrow v_{2E} \simeq 58.10^{-3}\,m/s$$

(6)





o momento linear é conservado na colisão

<u>em x</u>

$$P_{0x} = P_{fx}$$

$$P_{A0x} + P_{B0x} = P_{Afx} + P_{Bfx}$$

$$m_A v_A + 0 = (m_A + m_B)\, v \cos\phi \Rightarrow v = \frac{m_A v_A}{(m_A+m_B)\cos\phi}$$

<u>em y</u>

$$P_{0y} = P_{fy}$$

$$0 - m_B v_B = -(m_A + m_B)\, v \operatorname{sen}\phi \Rightarrow v = \frac{m_B v_B}{(m_A+m_B)\operatorname{sen}\phi}$$

$$\frac{m_A v_A}{\cos\phi} = \frac{m_B v_B}{\operatorname{sen}\phi}$$

$$\operatorname{tg}\phi = \frac{m_B v_B}{m_A v_A} = \frac{(1820\,\text{kg})(93{,}0\,\text{km/h})}{(1360\,\text{kg})(62{,}0\,\text{km/h})} = 2{,}00$$

$$\Rightarrow \phi = 63{,}5°$$

$$v = \frac{m_A \, v_A}{(m_A + m_B)\cos\phi}$$

$$v = \frac{(1360)(62,0)}{(1820 + 1360) \cdot (\cos 63,5°)}$$

$$v = 59,5 \ km/h$$

(8)

(a) conservação do momento linear

$$\sum \vec{P}_i = \sum \vec{P}_f$$

$$m_A \vec{v}_{iA} + m_B \vec{v}_{iB} = m_A \vec{v}_{fA} + m_B \vec{v}_{fB}$$

$$(2{,}0)(15\hat{i} + 30\hat{j}) + (3{,}0)(-10\hat{i} + 5{,}0\hat{j}) - (2{,}0)(-6{,}0\hat{i} + 30\hat{j}) = (3{,}0)\vec{v}_{fB}$$

$$(3{,}0)\vec{v}_{fB} = \cancel{30\hat{i}} + \cancel{60\hat{j}} - \cancel{30\hat{i}} + 15\hat{j} + 12\hat{i} - \cancel{60\hat{j}}$$

$$(3{,}0)\vec{v}_{fB} = +15\hat{j} + 12\hat{i}$$

$$\vec{v}_{fB} = +4\hat{i} + 5\hat{j}$$

(b) $\Delta K = K_f - K_i = \left(\frac{1}{2} m_A v_{Af}^2 + \frac{1}{2} m_B v_{Bf}^2\right) - \left(\frac{1}{2} m_A v_{Ai}^2 + \frac{1}{2} m_B v_{Bi}^2\right)$

$$v_{Ai}^2 = v_{Aix}^2 + v_{Aiy}^2 = (15)^2 + (30)^2 = 1125$$

$$v_{Bi}^2 = (-10)^2 + (5{,}0)^2 = 125$$

$$v_{Af}^2 = (-6)^2 + (30)^2 = 936$$

$$v_{Bi}^2 = (4)^2 + (5)^2 = 41$$

$$\Delta K = \frac{1}{2}\left[(2{,}0)(936) + (3{,}0)(41) - (2{,}0)(1125) - (3{,}0)(125)\right]$$

$$\Delta K = -315\ J$$