P2 FVV

Exercício 1

1) $f(x,y) = x^3 + 3xy^2 - 3y^3 - 9x \Rightarrow \begin{cases} f_x(x,y) = 3x^2 + 3y^2 - 9 = 0 \\ f_y(x,y) = 6xy - 9y^2 = 0 \end{cases} \Leftrightarrow \begin{cases} x^2 + y^2 = 3 \\ y(2x - 3y) = 0 \end{cases} \Leftrightarrow y = 0 \Rightarrow x = \pm 3$

$y \neq 0 \Rightarrow 2x = 3y \Leftrightarrow x = \frac{3}{2}y$

$\left(\frac{3}{2}y\right)^2 + y^2 = \frac{9}{4}y^2 + y^2 = 9 \Leftrightarrow$

Turma B P2

$\begin{cases} x^2 + y^2 = 3 \\ y(2x - 3y) = 0 \end{cases} \Leftrightarrow y = 0 \Rightarrow x = \pm\sqrt{3},$

$y \neq 0 \Leftrightarrow y = \frac{2}{3}x \Rightarrow x^2 + \frac{4}{9}x^2 = 3 \Rightarrow \frac{13}{9}x^2 = 3 \Rightarrow x^2 = \frac{3 \cdot 9}{13} \Rightarrow x = \pm\frac{3\sqrt{3}}{\sqrt{13}}$

$y = \frac{3\sqrt{3}}{\sqrt{13}} \cdot \frac{2}{3} = \frac{2\sqrt{3}}{\sqrt{13}}$, $P_0 = (\sqrt{3}, 0)$, $P_1 = (-\sqrt{3}, 0)$, $P_2 = \left(\frac{3\sqrt{3}}{\sqrt{13}}, \frac{2\sqrt{3}}{\sqrt{13}}\right)$, $P_3 = \left(-\frac{3\sqrt{3}}{\sqrt{13}}, -\frac{2\sqrt{3}}{\sqrt{13}}\right)$

Verificação: $\frac{3 \cdot 9 \cdot 3}{13} + \frac{3 \cdot 4 \cdot 3}{13} = 9\left(\frac{9+4}{13}\right) = 9$ OK.

$H(f)(x,y) = \begin{bmatrix} 6x, & 6y \\ 6y, & 6x - 18y \end{bmatrix} \Rightarrow \det(H(f)(\sqrt{3}, 0)) = \det\begin{bmatrix} 6\sqrt{3}, & 0 \\ 0, & 6\sqrt{3} \end{bmatrix} > 0 \Rightarrow P_0$ p.to de mín. loc.

$\det(H(f)(-\sqrt{3}, 0)) = \det\begin{bmatrix} -6\sqrt{3}, & 0 \\ 0, & -6\sqrt{3} \end{bmatrix} > 0 \Rightarrow P_1$ p.to de máx. loc.

$H(f)\left(\frac{3\sqrt{3}}{\sqrt{13}}, \frac{2\sqrt{3}}{\sqrt{13}}\right) = \begin{bmatrix} \frac{6 \cdot 3\sqrt{3}}{\sqrt{13}}, & \frac{2\sqrt{3} \cdot 6}{\sqrt{13}} \\ \frac{2 \cdot 6\sqrt{3}}{\sqrt{13}}, & \underbrace{\frac{6 \cdot 3\sqrt{3}}{\sqrt{13}} - 18 \cdot \frac{2\sqrt{3}}{\sqrt{13}}}_{= 0 - 18\frac{\sqrt{3}}{\sqrt{13}}} \end{bmatrix} \Rightarrow \det(H(f)(P_2)) = -18^2 \cdot \frac{3}{13} - (\ ) < 0 \Rightarrow P_2$ p.to sela

$\det(H(f)(P_3)) < 0 \Rightarrow P_3$ p.to de sela.

Exercício 2

$\binom{4}{2}=\frac{4!}{2!\,2!}=\frac{24}{4}=6$

2) $f(x,y)=x^4+2y^3-3xy$. $\Rightarrow f_{xxxx}(x,y)=\frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}(f(x,y))\right]\right]\right]=\frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}[4x^3-3y]\right]\right]=\frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}[12x^2]\right]=$

$=\frac{\partial}{\partial x}[24x]=24 \Rightarrow g(x,y)=x^4+2y^3-3xy-\frac{24}{24}(x-1)^4=x^4+2y^3-3xy-\left[x^4+4x^3(-1)+\binom{4}{2}x^2(-1)^2+\binom{4}{3}x(-1)^3+\binom{4}{4}(-1)^4\right]$

$=x^4+2y^3-3xy-x^4+4x^3-6x^2+4x-1=2y^3-3xy+4x^3-6x^2+4x-1=4x^3+2y^3-6x^2-3xy+4x-1=g(x,y)$

$g(x,y)+(x-1)^4=4x^3+2y^3-6x^2-3xy+4x-1+x^4-4x^3+6x^2-4x+1=x^4+2y^3-3xy$. Verificação O.K.

Método longo: $f(x,y)=x^4+2y^3-3xy \Rightarrow f(1,2)=1+2\cdot 8-3\cdot 2=1+16-6=11$, $f(0.98,2.01)=11.23209801$

$f_x(x,y)=4x^3-3y \Rightarrow f_x(1,2)=4-6=-2$; $f_y(x,y)=6y^2-3x \Rightarrow f_y(1,2)=6\cdot 4-3=21$; $f_{xx}(x,y)=12x^2 \Rightarrow f_{xx}(1,2)=12$; $f_{xy}(x,y)=-3 \Rightarrow f_{xy}(1,2)=-3$; $f_{yy}(x,y)=12y \Rightarrow f_{yy}(1,2)=24$

$f_{xxx}(x,y)=24x \Rightarrow f_{xxx}(1,2)=24$; $f_{xxy}(x,y)=0$, $f_{yyx}(x,y)=0$, $f_{yyy}(x,y)=12$. Logo $g(x,y)=f(1,2)+f_x(1,2)(x-1)+f_y(1,2)(y-2)+\frac{1}{2}\left[f_{xx}(1,2)(x-1)^2+2f_{xy}(1,2)(x-1)(y-2)+f_{yy}(1,2)(y-2)^2\right]$

$+\frac{1}{6}\left[f_{xxx}(1,2)(x-1)^3+3f_{xxy}(1,2)(x-1)^2(y-2)+3f_{yyx}(1,2)(x-1)(y-2)^2+f_{yyy}(1,2)(y-2)^3\right]=11-2(x-1)+21(y-2)+\frac{1}{2}\left[12(x-1)^2-6(x-1)(y-2)+24(y-2)^2\right]+$

$+\frac{1}{6}\left[24(x-1)^3+12(y-2)^3\right]=11-2x+2+21y-42+6(x^2-2x+1)-3(xy-2x-y+2)+12(y^2-4y+4)+4(x^3-3x^2+3x-1)+2(y^3-6y^2+12y-8)=$

$=11+2-42+6-6+48-4-16=11+4-16=-1$

$g(x,y)=-1-2x-12x+6x+12x+21y+3y-48y+24y+6x^2-12x^2-3xy+12y^2-12y^2+4x^3+2y^3$

$=-1+4x-6x^2-3xy+4x^3+2y^3$. $g(0.98,2.01)=11.232098$

Exercício 3

3) $f(x,y,z)=x+2y+z$, $x^2+y^2+z^2=16$. $\Rightarrow \mathcal{L}(x,y,z,\lambda)=x+2y+z-\lambda(x^2+y^2+z^2-16) \Rightarrow$

(*) $\begin{cases}\frac{\partial \mathcal{L}}{\partial x}(x,y,z,\lambda)=1-2\lambda x=0, \quad \lambda=0 \Rightarrow \nexists \\ \frac{\partial \mathcal{L}}{\partial y}(x,y,z,\lambda)=2-2\lambda y=0, \\ \frac{\partial \mathcal{L}}{\partial z}(x,y,z,\lambda)=1-2\lambda z=0, \\ \frac{\partial \mathcal{L}}{\partial \lambda}(x,y,z,\lambda)=0 \iff x^2+y^2+z^2=16.\end{cases}$

$\lambda\neq 0 \Rightarrow \begin{cases}2\lambda x=\lambda y, \\ 2\lambda x=2\lambda z, \\ x^2+y^2+z^2=16.\end{cases} \Rightarrow \begin{cases}y=2x, \\ z=x, \\ x^2+4x^2+x^2=6x^2=16 \iff 3x^2=8 \iff x=\pm\frac{2\sqrt{2}}{\sqrt{3}} \Rightarrow 1-2\lambda\frac{2\sqrt{2}}{\sqrt{3}}=0 \iff \lambda=\frac{\sqrt{3}}{4\sqrt{2}}\end{cases}$

$P_0=\left(\frac{2\sqrt{2}}{\sqrt{3}},\frac{4\sqrt{2}}{\sqrt{3}},\frac{2\sqrt{2}}{\sqrt{3}},\frac{\sqrt{3}}{4\sqrt{2}}\right)$, $P_1=\left(-\frac{2\sqrt{2}}{\sqrt{3}};\frac{4\sqrt{2}}{\sqrt{3}};\frac{2\sqrt{2}}{\sqrt{3}};\frac{\sqrt{3}}{4\sqrt{2}}\right)$. Verificação: Substituindo em (*)

$1-2\frac{\sqrt{3}}{4\sqrt{2}}\cdot\frac{2\sqrt{2}}{\sqrt{3}}=1-1=0$. O.K., $1-\frac{4\sqrt{2}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{4\sqrt{2}}=1-1=0$. O.K., $1-2\frac{\sqrt{3}}{4\sqrt{2}}\cdot\frac{2\sqrt{2}}{\sqrt{3}}=1-1=0$. O.K. $\left(\frac{2\sqrt{2}}{\sqrt{3}}\right)^2+\left(\frac{4\sqrt{2}}{\sqrt{3}}\right)^2+\left(\frac{2\sqrt{2}}{\sqrt{3}}\right)^2=$

$=\frac{8}{3}+\frac{32}{3}+\frac{8}{3}=\frac{48}{3}=16$.

$f(P_0)=\frac{2\sqrt{2}}{\sqrt{3}}+\frac{8\sqrt{2}}{\sqrt{3}}+\frac{2\sqrt{2}}{\sqrt{3}}=\frac{12\sqrt{2}}{\sqrt{3}}=4\sqrt{3}\sqrt{2}$, $f(P_1)=-4\sqrt{3}\sqrt{2} \Rightarrow$ P_0 é p.to de máx. abs.; P_1 é p.to de mín. absoluto.

$1=\int \frac{(3x-y)^2}{(x+y)}\,d$

Exercício 4

4) $I = \int_R \frac{(3x-y)^2}{e^{(x-y)}} dxdy$, $R = \{(x,y) \in \mathbb{R}^2 \mid 0 \leq x-y \leq 4, 2 \leq 3x-y \leq 8\}$

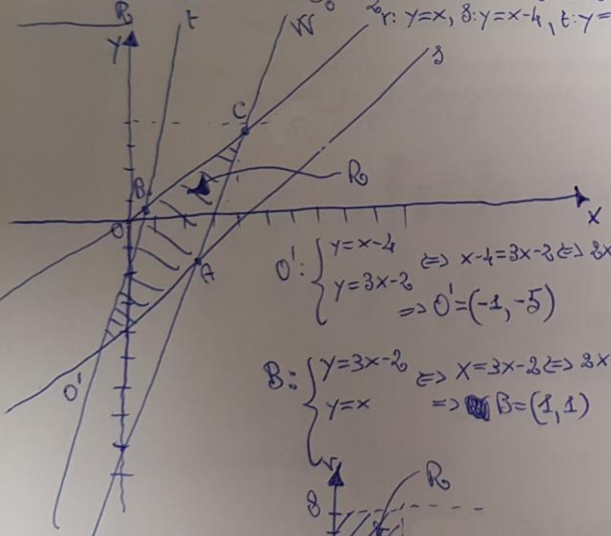
$\begin{cases} u = x-y \\ v = 3x-y \end{cases} \Rightarrow \tilde{R} = \{(u,v) \in \mathbb{R}^2 \mid 0 \leq u \leq 4, 2 \leq v \leq 8\}$, $\qquad -8+2^9 = 512-8 = 504$

$|du \wedge dv| = dudv = |(dx-dy) \wedge (3dx-dy)| = |-dx \wedge dy - 3dy \wedge dx| = |2dx \wedge dy| = 2dxdy \Rightarrow dxdy = \frac{1}{2}dudv$

$\Rightarrow I = \int_{\tilde{R}} \frac{v^2}{e^u} \cdot \frac{1}{2} dudv = \frac{1}{2}\int_0^4 \left\{ \int_2^8 \frac{v^2}{e^u} dv \right\} du = \frac{1}{2} \left\{ \int_0^4 e^{-u} du \right\} \left\{ \int_2^8 v^2 dv \right\} = \frac{1}{2} \left[-e^{-u}\right]_0^4 \left[\frac{v^3}{3}\right]_2^8 = \frac{1}{6}\left[-e^{-4}+1\right]\left[8^3-2^3\right] =$

$= \frac{504}{6}\left[1-e^{-4}\right] = \frac{252}{3}\left[1-e^{-4}\right] = 84\left[1-e^{-4}\right].$

$r: y=x$, $s: y=x-4$, $t: y=3x-2$, $w: y=3x-8$

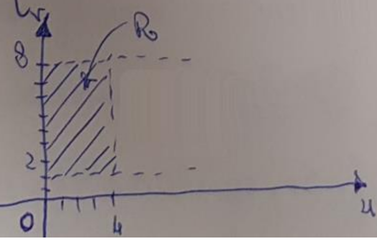


$O': \begin{cases} y = x-4 \\ y = 3x-2 \end{cases} \Leftrightarrow x-4 = 3x-2 \Leftrightarrow 2x = -2 \Leftrightarrow x = -1 \Rightarrow y = -5$, $\Rightarrow O' = (-1,-5)$

$A: \begin{cases} y = x-4 \\ y = 3x-8 \end{cases} \Leftrightarrow x-4 = 3x-8 \Leftrightarrow 2x = 4 \Leftrightarrow x = 2 \Rightarrow y = -2 \Rightarrow A = (2,-2)$

$B: \begin{cases} y = 3x-2 \\ y = x \end{cases} \Leftrightarrow x = 3x-2 \Leftrightarrow 2x = 2 \Leftrightarrow x = 1 \Rightarrow y = 1 \Rightarrow B = (1,1)$

$C: \begin{cases} y = 3x-8 \\ y = x \end{cases} \Leftrightarrow x = 3x-8 \Leftrightarrow 2x = 8 \Leftrightarrow x = 4 \Rightarrow y = 4 \Rightarrow C = (4,4).$



Exercício 5

5) $I = \int_R (x^2+y^2+z^2) e^{2\sqrt{x^2+y^2+z^2}} dxdydz$, onde $R = \{(x,y,z) \in \mathbb{R}^3 \mid x \geq 0, y \geq 0, z \geq 0, 0 \leq x^2+y^2+z^2 \leq 9\}$.

$\begin{cases} x = \rho \sin(\varphi)\cos(\theta) \\ y = \rho \sin(\varphi)\sin(\theta) \\ z = \rho \cos(\varphi) \end{cases}$

$\tilde{R} = \{(\rho,\varphi,\theta) \in \mathbb{R}^3 \mid 0 \leq \rho \leq 3, 0 \leq \varphi \leq \pi/2, 0 \leq \theta \leq \pi/2\}. \Rightarrow I = \int \rho^2 e^{2\rho} \rho^2 |\sin(\varphi)| d\rho d\varphi d\theta = \frac{\pi}{2} \left\{ \int_0^{\pi/2} \sin(\varphi) d\varphi \right\} \left\{ \int_0^3 \rho^4 e^{2\rho} \right\}$

$I = \frac{\pi}{2} \int_0^3 e^{2\rho} \rho^4 d\rho = \frac{\pi}{2} \left\{ \left[\frac{1}{2} e^{2\rho} \rho^4\right]_0^3 - 2\int_0^3 e^{2\rho} \rho^3 \right\} = \frac{\pi}{2} \left\{ \frac{81}{2} e^6 - \left\{ \left[e^{2\rho} \rho^3\right]_0^3 - 3\int_0^3 e^{2\rho} \rho^2 d\rho \right\} \right\} =$

$= \frac{\pi}{4} \left\{ 81e^6 - 2 \cdot 27e^6 + 6\int_0^3 e^{2\rho} \rho^2 d\rho \right\} = \frac{\pi}{4} \left\{ 27e^6 + 3 \left\{ \left[e^{2\rho} \rho^2\right]_0^3 - 2\int_0^3 e^{2\rho} \rho \, d\rho \right\} \right\} = \frac{\pi}{4} \left\{ 27e^6 + 27e^6 - 3\left\{ \left[e^{2\rho} \rho\right]_0^3 - \int_0^3 e^{2\rho} d\rho \right\} \right\} =$

$= \frac{\pi}{4} \left\{ 54e^6 - 9e^6 + \frac{3}{2}\left[e^{2\rho}\right]_0^3 \right\} = \frac{\pi}{4} \left\{ 45e^6 + \frac{3}{2}(e^6 - 1) \right\} = \frac{\pi}{8} \left\{ 90e^6 + 3e^6 - 3 \right\} = \frac{\pi}{8} \left\{ 93e^6 - 3 \right\} = \frac{3\pi}{8} \left\{ 31e^6 - 1 \right\}.$

Exercício 6

Turma B P2

5) $I = \int_{R} \sqrt{(x^2+y^2)^3}\,dxdydz$, onde $R = \{(x,y,z)\in\mathbb{R}^3 \mid 0 \le x^2+y^2+z^2 \le 4\}$. $\begin{cases} x=\rho\cos(\theta)\\ y=\rho\sin(\theta)\\ z=z\end{cases} \Rightarrow dxdydz = \rho\, d\rho\, d\theta\, dz$,

$\tilde{R} = \{(\rho,\theta,z)\in\mathbb{R}^3 \mid 0\le\rho^2+z^2\le 4,\ 0\le\rho\le 2,\ 0\le z\le 2,\ 0\le 2\pi\le\theta\}$. Logo $I = \int_{\tilde{R}} \rho^3 \rho\, d\rho\, d\theta\, dz = \int_{\tilde{R}} \rho^4\, d\rho\, d\theta\, dz =$

$= 2\pi \int_0^2 \rho^4 \left\{ \int_{-\sqrt{4-\rho^2}}^{\sqrt{4-\rho^2}} dz \right\} d\rho = 4\pi \int_0^2 \rho^4 \sqrt{4-\rho^2}\, d\rho = 8\pi^2.$

Resolver usando os integrais de Wallis, depois da substituição de variáveis $\rho = 2\sin(u)$.